Is Software Eating the World?[*]

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Abstract

The labor income share has fallen steadily since the 1980s in most advanced economies. The most widely accepted explanations of this phenomenon evolve around the substitution between capital and labor, but micro-level estimates more often than not show that capital and labor are *complements*. Using firm- and establishment-level data from Korea, we divide capital into equipment and software. Our estimated elasticities of substitution show that equipment and labor are complements (0.5), consistent with other micro-level estimates, but software and labor are substitutes (1.7), a novel finding that reconciles conflicting views on the elasticities in the literature. As the quality of software improves, the labor share falls within firms. In addition, production reallocates to those firms that use software more intensively, as they become relatively more productive. It turns out these firms tend to have low labor shares (an empirical question, since in theory software-intensive firms may have above-average labor shares and below-average equipment shares), and hence the reallocation further reduces the aggregate labor share. Software, not equipment capital, is the key to the decline of the labor income share.

Keywords: Labor income share, elasticity of substitution, software-embodied technological change, reallocation.

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Software is eating the world.

— Marc Andreessen (The Wall Street Journal, August 20, 2011)

1 Introduction

The labor share remained more or less constant over time for much of the 20th century. For example, Keynes (1939) wrote that "the stability of the proportion of the national dividend accruing to labour" was "one of the most surprising, yet best-established, facts, both for Great Britain and for the United States." However, the labor share started to show a downward trend in the US and other advanced economies since the 1980s. With the heightened interest in economic inequality after the financial crisis of 2007–08, economists have raised a variety of explanations regarding the decline of the labor share. Grossman and Oberfield (2022) gives a thorough review of the literature.

Three leading explanations have emerged. The first is that capital and labor are substitutes, and the more efficient production of capital goods decreased the labor share over time (Karabarbounis and Neiman, 2013). Second, the growing importance of intangible capital and systemic mismeasurement of labor's contribution to intangible capital resulted in lower labor share in the data (Koh et al., 2020). Third, the reallocation of production toward large firms with low labor shares and fast-growing firms with falling labor shares reduced the aggregate labor share (Autor et al., 2020; Kehrig and Vincent, 2021)

This paper contributes to this debate in two ways. First, we reconcile the tension between the first explanation above, which requires an elasticity of substitution between capital and labor larger than one (e.g., Karabarbounis and Neiman, 2013; Hubmer, 2021), and the micro-level estimates of this elasticity that are less than one found many times over (Antras, 2004; Raval, 2019; Oberfield and Raval, 2021, among others). Second, we clarify the connection among these three explanations via a common causal factor: the rise of software.

The price of capital has declined for decades, because of the fast productivity growth of the capital-producing sectors (known as capital-embodied technological change). What is less known is that the measured quality of software in the National Accounts has improved faster than that of equipment—i.e., software price has fallen faster than equipment price.

Using firm-level and establishment-level data from Korea, we divide capital into equipment (machinery) and software. This is possible because Korean firms keep track of software investment to comply with the local accounting standards. Extending the approach of Oberfield and Raval (2021) to our environment with three factors of production (labor, equipment, and software), we estimate the elasticity of substitution between labor and either type of capital by instrumenting for wage variations across regions, while assuming that the price of equipment and software is similar across regions.

Our estimation shows that the elasticity of substitution between labor and equipment is less than one (0.5), consistent with the micro-level estimates from the US and some other countries, which implies that the substitution between equipment and labor is not the source of the declining labor income share. Our novel finding is that the elasticity of substitution between labor and software is greater than one (1.7), implying that the rise of software is the reason for the decline in the labor income share.

A change in factor prices not only changes the factor income shares within a firm, but also reallocates resources across firms that are heterogeneous in terms of factor intensity. The "macro" elasticity that takes into account such equilibrium effects will differ from the micro elasticity (Oberfield and Raval, 2021). An elasticity of substitution between software and labor that exceeds one implies that firms using software more intensively see a larger reduction in labor share than others. Such firms also become effectively more productive than others and hence become larger, if the demand elasticity across differentiated products is also larger than one. Unlike in two-factor models, with three factors, whether such reallocation from low software share firms to high software share firms further reduces the aggregate labor income share is an empirical question. It is possible that high software share firms have above-average labor shares and below-average equipment shares, in which case this reallocation channel will raise the aggregate labor income share. In the data, the correlation between a firm's software share and its labor share is negative, which means that the reallocation channel further reduces the aggregate labor income share.

Quantitatively, the decline in software price explains about half of the labor share decline in Korea between 1976 and 2019. The reallocation channel accounts for 31 percent of the fall in the labor share in the model, with the within-firm channel accounting for the remaining 69 percent. The fall in equipment price, on the other hand, pushes up the labor share, since equipment and labor are complements.

In the Korean data, we also document that a firm's software intensity correlates positively with (i) a decrease in (within-firm) labor share, (ii) sales growth, (iii) an increase in markup, and (iv) faster total factor productivity (TFP) growth. All these patterns are consistent with the conclusion that software-embodied technological change has played a key role in the decline of the aggregate labor income share. At the industry level, we find that industries with relatively higher ratios of software capital to value-added are more concentrated.

In relation to the literature, we clarify that it is software not equipment that substitutes labor and reduces the labor income share, and reconcile the disagreement among previous estimates on the capital-labor substitution elasticities. Our emphasis on software also supports the view that highlights the role of intangibles, of which software is one of the better measured components. Moreover, we capture the equilibrium effect and the reallocation across firms, which accords with the recent work emphasizing reallocation to low labor share firms or firms with falling labor shares. Overall, our analysis implies that it is crucial to distinguish between capital types when looking at the elasticity of substitution between labor and capital, which is an essential parameter in understanding the labor share dynamics over long periods.

The rest of the paper is structured as follows. We document empirical evidence on the relationship between a firm's software intensity and its other variables in Section 2. In Section 3, we introduce a model framework that relates software-embodied technological change to the observed patterns in the data. We show how software-embodied technological change interacts with the elasticity of substitution between labor and capital to determine the direction and magnitude of factor income share changes. We then estimate the micro-elasticity and aggregate it into macro-elasticity in Section 4. In Section 5, we assess the contribution of software-embodied technological change to the decline in the aggregate labor income share. Section 6 concludes.

2 Motivating Facts

2.1 Labor Income and Capital Income

We begin by documenting the evolution of labor income and capital income by capital type in Korea. Figure 1 plots the aggregate labor income share between 1975 and 2019. Our measure of the labor income share is compensation of employee plus the labor income of the proprietors as a share of gross domestic product, where the labor income of the proprietors is estimated by assuming that the labor income share of the proprietors' income is the same as that in the rest of the economy (Gollin, 2002).[1] The labor share shows a declining trend, with the most pronounced decline between the mid 1990s and the early 2000s.

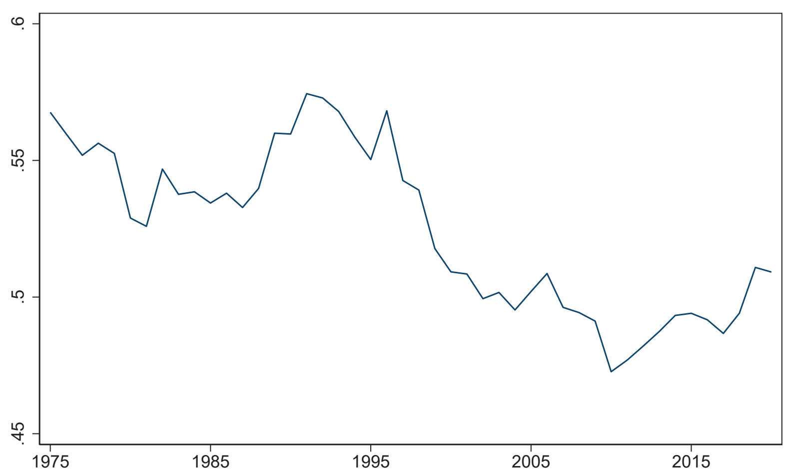

Fig. 1: Labor share

The denominator of the labor share consists of labor income, capital income, and profits. To be specific, the labor share is

$$LS = \frac{wL}{\mu(wL + \sum_j R^j K^j)}, \tag{1}$$

where wL is labor income, μ is the aggregate markup, and $R^j K^j$ is income of capital type j. To compute capital income, we need an estimate on R^j for each capital type j.

[1] To be specific, we first compute the net labor share $NLS = \frac{CE+NLS\times PI}{GDP-CFC}$, where NLS is the net labor share, CE is the compensation of employees, PI is proprietors' income, CFC is the consumption of fixed capital (depreciation), and GDP is the gross domestic product. Then the total labor income in the aggregate economy is $wL = CE + NLS \times PI$, and the aggregate labor share is $LS = wL/GDP$. Since the Korean National Accounts do not report the proprietors' income before 2009, we use the operational surplus in the household sector as proxy for proprietors' income.

For this purpose, we assume that the gross rate of return on capital R^j satisfies

$$R^j = (1+r)p^j_{t-1} - (1-\delta^j)p^j_t, \tag{2}$$

where r is the net rate of return, p^j is the price of capital j (in period $t-1$ and t), and δ^j is the depreciation rate of capital j.

We first get LS, wL, K^j, p^j, and δ^j from the National Accounts. Specifically, wL is the compensation of employees plus the labor income of the self proprietors. p^j is the price index of investment of capital type j divided by the price index of consumption expenditure, and K^j is the net capital stock of capital type j. We estimate δ^j from the investment minus the change in the net capital stock, divided by the previous period's capital stock ($\delta^j = (K^j_{t-1} + I^j_t - K^j_t)/K^j_{t-1}$). For the aggregate markup, we estimate μ following De Loecker et al. (2020) from the firm-level data—see Appendix B for details. We then impute the rate of return on capital from equations (1) and (2).

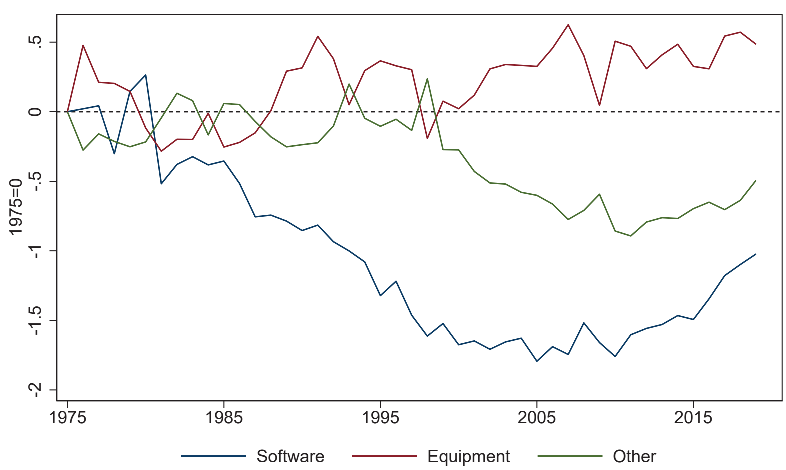

Fig. 2: Labor income relative to capital income by type of capital

In Figure 2, we compare log $\frac{wL}{R^jK^j}$ across capital type j's. By construction, the labor income share declines when the capital income grows faster than the labor income. Figure 2 demonstrates that the pattern is not the same across capital types. In particular, software income has shown the fastest growth, whereas equipment income has even decreased relative to labor income. Moreover, the growth of software income has been concentrated in the 1990s and the early 2000s, when the labor share declined the most in Figure 1.

2.2 Firm-Level Empirics

2.2.1 Data

For the firm-level analysis in this section, we use the data from the KISDATA database between 2000 and 2018. KISDATA reports financial information for firms listed on the Korea Stock Exchange and those unlisted firms required to publish external auditing reports.[2] We exclude financial firms and quasi-governmental and non-profit firms from the sample. Comparing our sample with the National Accounts, our KISDATA sample covers about 47 percent and 56 percent of the compensation of employees and the operational surplus in the entire non-financial corporate sector in 2018.

Construction of the labor share requires labor compensation and value added. We combine employee compensation and benefits in the income statement and the labor cost in the manufacturing cost statement to obtain the labor compensation. We get value added by adding up operational profit, depreciation and amortization, taxes and dues, and labor compensation.

We use the variable "intangible asset - software" in firms' balance sheet to measure their software asset. A firm classifies its software purchases from outside as software assets (in intangible assets) according to Korean Generally Accepted Accounting Principles (K-GAAP). Detailed explanation for the data is in the Appendix B.

2.2.2 Software Intensity: Panel Regressions

We begin by documenting that firms' software intensity is associated with a decline in within-firm labor share and also with faster sales growth. Our regression equation is as follows.

$$y_{i,t} = \gamma_i + \delta_t + \beta_s \times s_{i,t-1} + \beta_e \times e_{i,t-1} + \varepsilon_{i,t}, \tag{3}$$

where $y_{i,t}$ is the variable of interest (e.g., a change in the labor share or sales) at time t, $s_{i,t-1}$ is a firm i's software intensity measured by software asset divided by its value added at time $t-1$, and $e_{i,t-1}$ is a firm i's equipment intensity measured by equipment asset divided by its value added at time $t-1$.

[2] Detailed criteria for external auditing requirement varies over time. Until 2008, firms whose asset value exceeded 7 billion KRW had to be audited externally. Since 2009 (2014), (i) firms with asset value greater than 10 billion (12 billion) KRW, (ii) asset value greater than 7 billion KRW and liability greater than 7 billion KRW, or (iii) asset value greater than 7 billion KRW and the number of employees more than 300 were subject to external auditing.

	Δ labor share	Δ log sales
$s_{i,t-1}$	-1.122***	1.473***
	(0.132)	(0.329)
$e_{i,t-1}$	-0.008	0.028*
	(0.006)	(0.014)
Obs.	42,225	42,217
R^2	0.191	0.289

Table 1: Software Intensity and Changes in Labor Share and Sales at the Firm Level

Standard errors are clustered at the two-digit industry level. *, **, and *** indicate significance at the 10, 5, and 1 percent level, respectively.

The estimation results are in Table 1. Firms' software intensity correlates with a faster decline of their labor share and a higher sales growth. The results suggest that a more intense use of software could lead to a decline in the aggregate labor share, not only through within-firm labor share declines, but also through a between-firm reallocation of their sales shares. More importantly, it implies that software intensity can be a key factor behind the recent finding by Kehrig and Vincent (2021) that the decline of the aggregate labor share in the US manufacturing sector is mostly accounted for by those establishments whose labor share falls and sales increase at the same time. On the other hand, the equipment intensity does not show a significant relation with a change in the labor share although it has a weak positive relationship with a sales growth.

In addition, software intensity may be related to firms' productivity growth. We estimate firm-level TFP following Olley and Pakes (1996) utilizing the adjustment suggested by Ackerberg et al. (2015). We find that software intensity is positively associated with firms' TFP growth as can be seen in Table 2. Equipment intensity also shows a positive relationship with the TFP growth, but the relationship was much less tight.

Does the productivity growth get passed through to consumers or simply translate into higher markups? To answer this question, we follow Baqaee and Farhi (2019) and measure firms's markups using three different methods: (i) the user cost approach (μ^{UC}), (ii) the production function estimation approach (μ^{PF}), and (iii) the accounting profits approach (μ^{AP}). The first method measures markups by comparing total costs to sales, where total costs include the user cost of capital. The second method mea-

	$\Delta \ln \text{TFP}_{i,t}$
$s_{i,t-1}$	0.897^{**}
	(0.390)
$e_{i,t-1}$	0.033^{*}
	(0.019)
Obs.	16,868
R^2	0.300

Table 2: Software Intensity and TFP growth at the Firm Level

Standard errors are clustered at the two-digit industry level. *, **, and *** indicate significance at the 10, 5, and 1 percent level, respectively.

sures markups by computing the ratio between the elasticity of the production function to variable input and the share of variable input in revenues. The elasticity of the production function to variable input comes from the production function estimation, as in De Loecker et al. (2020). Lastly, the accounting profit approach define markups as sales divided by sales minus operating profit. The details on the measurement of markups are in Appendix B.

	$\Delta \log \mu_{i,t}^{UC}$	$\Delta \log \mu_{i,t}^{PF}$	$\Delta \log \mu_{i,t}^{AP}$
$s_{i,t-1}$	0.287^{***}	0.344^{***}	0.388^{***}
	(0.085)	(0.074)	(0.088)
$e_{i,t-1}$	0.022^{**}	0.017^{***}	0.003^{*}
	(0.009)	(0.005)	(0.002)
Obs.	36,757	38,369	40,762
R^2	0.250	0.246	0.248

Table 3: Software Intensity and Markups

Standard errors are clustered at the two-digit industry level. *, **, and *** indicate significance at the 10, 5, and 1 percent level, respectively.

Each method has pros and cons, and captures conceptually different objects. Despite their differences, software intensity shows a significant correlation with increasing markups across all three measures (Table 3). Note that the increase in markups implies the decline in the labor share, ceteris paribus. Again, the equipment intensity is also positively associated with increasing markups, but the relation is less tight.

Finally, turning to an industry-level analysis, we ask whether the software intensity of an industry is correlated with market concentration. Such a correlation is possible if larger firms have disproportionately higher software intensity. We compute the Herfindahl-Hirschman Index (HHI), concentration ratios (CR4 and CR8) for each two-digit industry, and estimate following:

$$\text{concentration}_{j,t} = \alpha + \beta_s \times s_{j,t} + \beta_e \times e_{j,t} + \varepsilon_{j,t}, \tag{4}$$

where concentration$_{j,t}$ is the concentration measure of industry j in year t, and $s_{j,t}$ and $e_{j,t}$ is the total software and equipment stock divided by the value added of industry j in year t, respectively. The estimation results in Table 4 show that higher software intensity correlates with higher market concentration at the industry level. For the concentatration measures, equipment intensity also shows a significant and positive relations as well.

	HHI	CR4	CR8
$s_{j,t}$	0.747***	0.384***	0.353***
	(0.000)	(0.000)	(0.000)
$e_{j,t}$	0.058***	0.045***	0.031***
	(0.000)	(0.000)	(0.000)
Obs.	1,143	1,143	1,143
R^2	0.035	0.019	0.016

Table 4: Software Intensity and Market Concentration at the Industry Level

Robust standard errors are in parentheses. *, **, and *** indicate significance at the 10, 5, and 1 percent level, respectively.

To summarize, the empirical evidence from the firm-level data in Korea suggests that software could be a factor that explains the decline of the labor income share. A firm with higher software intensity has experienced a faster labor share decline, a higher sales growth, and a rise in markups. Moreover, industries that use software more intensively tend to have a higher market concentration. All these observations suggest that the role of software merits a separate investigation, which we now turn to.

3 Model

3.1 Production Function

To investigate a separate role of software in shaping factor income shares, we consider a production function that has software as a separate input from equipment.

Specifically, we define a production at firm i as

$$Y_i = \left[\left(\alpha_i^L (A_L L_i)^{\frac{\sigma_e - 1}{\sigma_e}} + \alpha_i^K (A_K K_i)^{\frac{\sigma_e - 1}{\sigma_e}} \right)^{\frac{\sigma_e(\sigma_S - 1)}{(\sigma_e - 1)\sigma_S}} + \alpha_i^S (A_S S_i)^{\frac{\sigma_S - 1}{\sigma_S}} \right]^{\frac{\sigma_S}{\sigma_S - 1}}, \tag{5}$$

where Y_i is value added, L_i is labor, K_i is tangible capital (or equipment), and S_i is software stock of firm i. α_i^L, α_i^K, and α_i^S are intensity of labor, tangible capital, and software in firm i's production technology, respectively. A_L, A_K, and A_S represent economy-wide factor-augmenting technologies for labor, equipment, and software, respectively. For notational convenience, we also define the equipment-labor bundle X_i as follows.

$$X_i = \left(\alpha_i^L (A_L L_i)^{\frac{\sigma_e - 1}{\sigma_e}} + \alpha_i^K (A_K K_i)^{\frac{\sigma_e - 1}{\sigma_e}} \right)^{\frac{\sigma_e}{\sigma_e - 1}}. \tag{6}$$

Our production function (5) has two important advantages in investigating the role of software. First, two different parameters govern the elasticity of substitution between labor and equipment (σ_e) and the elasticity of substitution between labor and software (σ_s). Therefore, it can capture different labor share responses to technological changes embodied in different types of capital. Second, all firms are different in how intensively they use each factor, as captured by α_i's. Therefore, how much factor shares change in response to a factor-augmenting technological change is also different across firms. This implies that aggregate changes in the factor income share depend not only on within-firm adjustments but also on the composition changes across firms. In other words, the elasticity of substitution at the aggregate level will be different from the elasticity of substitution at the firm level (Oberfield and Raval, 2021).

An aggregate production function is defined as

$$Y = \left(\sum_i \gamma_i Y_i^{\frac{\epsilon - 1}{\epsilon}} \right)^{\frac{\epsilon}{\epsilon - 1}}, \tag{7}$$

where Y_i is a firm-level value added production in equation (5). Note that the aggregate production function (7) implies the demand for Y_i given by equation (8),

$$Y_i = \gamma_i^{\epsilon} \left(\frac{p_i}{p} \right)^{-\epsilon} Y, \tag{8}$$

where $p = \left(\sum_i \gamma_i^{\epsilon} p_i^{1-\epsilon} \right)^{\frac{1}{1-\epsilon}}$ is the ideal price index.

Firm i takes the factor price of labor (w), tangible capital (r), and software (q) as given, and maximizes the profit:

$$\max_{p_i, Y_i, K_i, S_i} p_i Y_i - wL_i - rK_i - qS_i,$$

subject to equations (5) and (8). The solution to the problem satisfies the following FOCs:

$$w = p_i \alpha_i^L A_L^{\sigma_e - 1} \sigma_e \left(\frac{Y_i}{X_i} \right)^{\frac{1}{\sigma_s}} \left(\frac{X_i}{L_i} \right)^{\frac{1}{\sigma_e}}, \tag{9}$$

$$r = p_i \alpha_i^K A_K^{\sigma_e - 1} \sigma_e \left(\frac{Y_i}{X_i} \right)^{\frac{1}{\sigma_s}} \left(\frac{X_i}{K_i} \right)^{\frac{1}{\sigma_e}}, \tag{10}$$

$$q = p_i \alpha_i^S A_S^{\sigma_s - 1} \sigma_s \left(\frac{Y_i}{S_i} \right)^{\frac{1}{\sigma_s}}, \tag{11}$$

$$p_i = \frac{\epsilon}{\epsilon - 1} \left[\phi_i^{1-\sigma_s} + \alpha_i^{S^{\sigma_s}} \left(\frac{q}{A_S} \right)^{1-\sigma_s} \right]^{\frac{1}{1-\sigma_s}}. \tag{12}$$

Note that we introduce ϕ_i, the price of the tangible capital-labor bundle X_i, for notational convenience.

$$\phi_i := \left(\alpha_i^{L^{\sigma}}{}_e \left(\frac{w}{A_L} \right)^{1-\sigma_e} + \alpha_i^{K^{\sigma}}{}_e \left(\frac{r}{A_K} \right)^{1-\sigma_e} \right)^{\frac{1}{1-\sigma_e}}. \tag{13}$$

Lastly, the factor market clearing conditions are

$$L = \sum_i L_i, \; K = \sum_i K_i, \text{ and } S = \sum_i S_i. \tag{14}$$

Given L, K, and S, we can solve the model from equation (5), (7), (8), (9), (10), (11), (12), and (14).

3.2 Elasticity of Substitution

We now establish the changes in factor shares in response to capital-augmenting technological changes or exogenous changes in factor prices, which can be captured by the elasticity of substitution (Stern, 2011). As Grossman and Oberfield (2022) pointed out, the elasticity of substitution is essential in understanding the decline of the aggregate labor share, no matter what the proposed mechanism is, because any changes in the economic environment would also affect prices through the general equilibrium effect.

One problem when considering software as a separate input from the traditional capital is that there are multiple ways to define the elasticity of substitution when there are more than two inputs in the production function. For example, the elasticity of substitution between labor and equipment will change, depending on whether we fix output and software altogether or only output and allow the software to vary. Hence, we need to clarify first what we call the elasticity of substitution between factors.

Rather than discuss all different ways of defining the elasticity of substitution, we focus on the simplest one. Here, we use the so-called Allen-Uzawa elasticity. The Allen-Uzawa elasticity of substitution between factor x and y is defined as

$$\varepsilon_{x,y} = \frac{CC_{xy}}{C_x C_y},$$

where C is the optimal cost function and C_x is the partial derivative of the optimal cost function with respect to a change in the price of input x while fixing all other prices constant. In our framework, the Allen-Uzawa elasticity of substitution between labor and tangible capital or software is simply σ_e or σ_s.

Now we can show the following.

Proposition 1 (Firm-level elasticity of substitution) *σ_e and σ_s satisfy the following.*

$$\sigma_e - 1 = \frac{d \ln rK_i / wL_i}{d \ln w/r} = \frac{d \ln k_i / (1-k_i)}{d \ln w/r}, \tag{15}$$

$$\sigma_s - 1 = \frac{d \ln qS_i / (wL_i + rK_i)}{(1-k_i) d ln w/q + k_i d \ln r/q} = \frac{d \ln s_i / (1-s_i)}{d \ln \phi_i / q}. \tag{16}$$

Proof *In Appendix A.* ∎

Note that we introduced several notations that represent factor shares at the firm level for convenience. Specifically,

$$k_i := \frac{rK_i}{wL_i + rK_i}, \; s_i := \frac{qS_i}{wL_i + rK_i + qS_i},$$

$$\ell_i := \frac{wL_i}{wL_i + rK_i + qS_i}, \; \kappa_i := \frac{rK_i}{wL_i + rK_i + qS_i},$$

where k_i is equipment income share in the equipment-labor bundle X_i, and s_i is software income share in value-added, ℓ_i is labor income share in value-added, and κ_i is equipment income share in value added.

Proposition 1 establishes that the direction of changes in the factor income shares in response to an exogenous change in the input price depends on whether σ_e or σ_s is greater than one or not. For example, when the price of equipment (r) falls, the ratio of labor income to equipment income falls when $\sigma_e > 1$ and rises when $\sigma_e < 1$. Similarly, when the price of software (q) falls, the share of software income compared to labor and equipment income rises when $\sigma_s > 1$ and falls when $\sigma_s < 1$.

We can easily obtain the following corollary that an exogenous change in the input price mirrors a factor-augmenting technological change.

Corollary 1 *σ_e and σ_s satisfy the following.*

$$\sigma_e - 1 = \frac{d \ln rK_i/wL_i}{d \ln A_K/A_L} = \frac{d \ln k_i/(1-k_i)}{d \ln A_K/A_L}, \tag{17}$$

$$\sigma_s - 1 = \frac{d \ln qS_i/(wL_i + rK_i)}{(1-k_i) dln A_S/A_L + k_i d \ln A_S/A_K} = \frac{d \ln s_i/(1-s_i)}{d \ln \phi_i A_S}. \tag{18}$$

Proof *In Appendix A.* ∎

Corollary 1 shows that factor-augmenting technological changes are equivalent to the decrease in the price of the same factor. In other words, we cannot separately identify changes in the factor-augmenting technologies from changes in factor prices. In this context, it is difficult to estimate σ from time series, as factor biased technological changes are typically unobservable.

Since we do not restrict factor distribution across firms (or α_i's), the model does not allow a well-defined aggregate production function with aggregate capital and labor. However, as Oberfield and Raval (2021) showed, we can still derive the relationship between the firm-level elasticity of substitution in Proposition 1 and changes in the aggregate factor income shares in response to changes in factor prices. To see this, we define the aggregate elasticity of substitution that captures the response of the aggregate factor shares to changes in factor prices.

Definition 1 (Aggregate elasticity of substitution) *The aggregate elasticity of substitutions $\bar{\sigma}_e^w$, $\bar{\sigma}_e^r$, $\bar{\sigma}_s^w$, $\bar{\sigma}_s^r$, and $\bar{\sigma}_s^q$ are*

$$\bar{\sigma}_e^w - 1 := \frac{d \ln rK/wL}{d \ln w} \tag{19}$$

$$\bar{\sigma}_e^r - 1 := -\frac{d \ln rK/wL}{d \ln r} \tag{20}$$

$$\bar{\sigma}_s^w - 1 := \frac{d \ln qS/(wL + rK)}{wL/(wL + rK) \times d \ln w} \tag{21}$$

$$\bar{\sigma}_s^r - 1 := \frac{d \ln qS/(wL + rK)}{rK/(wL + rK) \times d \ln r} \tag{22}$$

$$\bar{\sigma}_s^q - 1 := -\frac{d \ln qS/(wL + rK)}{d \ln q} \tag{23}$$

Different from Oberfield and Raval (2021), we need to define the aggregate elasticity of substitution between factors for each input price w, r, and q. In other words, even when a change in the relative wage to capital price is the same, the corresponding change in the ratio of labor income to capital income will be different, depending on whether the wage increased or the price of capital fell.

Intuitively, because the labor income share is not always equal to one minus the equipment income share with three inputs (i.e., $\ell_i \neq 1 - \kappa_i$), the changes in the aggregate labor share resulting from the change in wage or the change in capital price can be different. For example, a firm with low ℓ_i can also feature low κ_i, when s_i is large. If this is the case, firms that benefited more from a wage increase do not necessarily benefit more from a fall in capital price. This would not be the case with two factors, as then firms' labor income share is perfectly negatively correlated with their capital income share.[3]

Now we derive the main proposition that link the firm-level elasticity of substitution to the aggregate elasticity of substitution. As before, we introduce several notations for convenience:

$$k := \frac{rK}{wL + rK}, \; s := \frac{qS}{wL + rK + qS},$$
$$\ell := \frac{wL}{wL + rK + qS}, \; \kappa := \frac{rK}{wL + rK + qS},$$
$$\theta_i := \frac{wL_i + rK_i}{wL + rK}, \; \omega_i := \frac{wL_i + rK_i + qS_i}{wL + rK + qS},$$

where k is aggregate income share of equipment to the income share of the equipment-labor composite, s is aggregate software income share, ℓ is aggregate labor income

[3] Oberfield and Raval (2021) has material input as another factor in the production function, but they still obtain a symmetry. The reason is that they assume that a change in the relative price of the material to capital is always proportional to a change in the wage relative to the price of capital. Their assumption implies that material is produced by combining labor and capital (in a Cobb-Douglas manner), and so there are still only two factors of production. We can not make the same assumption because our third input, software, is an independent factor of production.

share, κ is aggregate equipment income share, θ_i is firm i's share of the labor-equipment income, and ω_i is firm i's share of the aggregate value-added. Then we have the following proposition.

Proposition 2 (Aggregation) *The aggregate elasticities of substitution satisfy*

$$\bar{\sigma}_e^w = (1-\chi)\sigma_e + \chi\zeta^w\sigma_s + \chi(1-\zeta^w)\epsilon, \tag{24}$$

$$\bar{\sigma}_e^r = (1-\chi)\sigma_e + \chi\zeta^r\sigma_s + \chi(1-\zeta^r)\epsilon, \tag{25}$$

$$\bar{\sigma}_s^q = (1-\xi^r)\sigma_s + \xi^q\epsilon, \tag{26}$$

$$\bar{\sigma}_s^w = (1-\xi^w)\sigma_s + \xi^w\epsilon, \tag{27}$$

$$\bar{\sigma}_s^r = (1-\xi^r)\sigma_s + \xi^r\epsilon, \tag{28}$$

where, $\chi := \frac{\sum_i (k_i-k)^2\theta_i}{k(1-k)}$, $\xi^w := -\frac{\sum_i \omega_i(s_i-s)(\ell_i-\ell)}{s\ell}$, $\xi^r := -\frac{\sum_i \omega_i(s_i-s)(\kappa_i-\kappa)}{s\kappa}$, $\xi^q := \frac{\sum_i \omega_i(s_i-s)^2}{s(1-s)}$, $\zeta^w := \frac{\sum_i (k_i-k)(1-k_i)\theta_i s_i}{\sum_i (k_i-k)(1-k_i)\theta_i}$, *and* $\zeta^r := \frac{\sum_i (k_i-k)k_i\theta_i s_i}{\sum_i (k_i-k)k_i\theta_i}$.

Proof *In Appendix A.* ∎

The aggregate elasticity of substitution is a weighted average of the firm-level elasticity of substitution and other distributional moments that govern between-firm reallocation. The aggregate elasticity of substitution between labor and equipment, for example, is a weighted average of σ_e, σ_s, and ϵ, with ϵ being the elasticity of substitution across differentiated products. The weight parameter χ is proportional to the variance of k_i, meaning that it gets larger as k_i's are more dispersed. Intuitively, as k_i's are more dispersed, the reallocation across firms is more important than within-firm adjustment $(1-\chi)\sigma_e$. When there is no difference in the equipment intensity k_i, for example, all firms will adjust the factor income ratio by the same proportion. Hence the aggregate elasticity would be the same as the firm-level elasticity in this case.

The between-firm reallocation also depends on σ_s and ϵ. Following the change in input price, a firm's share of the equipment-labor bundle in the aggregate economy will change (depending on ϵ) and also because it substitutes its production toward or away from software (depending on σ_s). Note that the importance of the latter depends on how much software each firm uses, represented by s_i in ζ.

We now discuss the aggregate elasticity of substitution between software and labor or equipment, which is our unique feature. The parameter σ_s in the production function (5) captures the elasticity of substitution between software and the equipment-labor bundle as a whole. Therefore, the aggregate elasticity of substitution between software and equipment or labor does not depend on σ_e. However, the changes in the software income share still depend on whether the shock is from the wage, the price of equipment, or the price of software, as shown in equation (26), (27), and (28).

Suppose that the price of equipment goes down. Then the price of the equipment-labor composite will go down, and there will be substitution away from software. However, the magnitude of the decline in the bundle price depends on how intensively a firm uses its equipment. As a firm with a higher κ_i will experience more drop in the composite price ϕ_i, the firm level substitution will be more prominent in a firm with a higher κ_i. Therefore, the overall importance of firm-level substitution away from software will be smaller (i.e., higher ξ_r) when a firm with a higher κ_i tends to have a lower s_i (i.e., negative covariance between κ_i and s_i). This is why ξ^r is proportional to the minus of the covariance between κ_i and s_i.

Following the same logic, ξ^w is proportional to the minus of the covariance between ℓ_i and s_i. Importantly, a smaller s_i does not necessarily imply a bigger ℓ_i or κ_i because we do not restrict the factor income distribution and because we have three inputs. For example, suppose the covariance term between κ_i and s_i is positive in the data. In that case, the reallocation responding to capital-specific technological change will go in the direction opposite to the within-firm adjustment.

Lastly, the relative changes in the software income share depend more on between-firm reallocation when s_i's are more dispersed as ξ^q is proportional to the variance of s_i. Moreover, considering that ϵ is greater than one under any reasonable parameterization, between-firm reallocation would lead to an increase in the aggregate software income share (or a fall in the labor share, ceteris paribus) in response to a decline in the software price (equation (23)).

In Section 2, we documented that firms with higher s_i tend to experience a faster decline in labor share and a higher sales growth. Our model is consistent with these observations if (i) $\sigma_s > 1$ (within-firm adjustment), (ii) $\epsilon > 1$ (between-firm reallocation), and (iii) the main source of exogenous variations are software-embodied technological change or a fall in the price of software. It is natural that $\epsilon > 1$. It is also well known that the price of capital goods has experienced a faster decline than the price of consumption goods, often referred to as investment-specific technological change. Though less known, the price of software has declined even faster than the price of equipment. We have yet to find out whether σ_s is greater than one or not, which we examine in the next section.

4 Estimation Results

The elasticity of substitution between labor and capital is difficult to estimate because the relative factor inputs and their relative prices are determined simultaneously. As implied by Proposition 1 and Corollary 1, the elasticity of substitution is not identified separately from the factor bias of technological change. To estimate the elasticity of substitution from the observed factor income shares and relative factor prices, one needs an instrument for the relative prices orthogonal to the biased technological changes.

Using macro-level (time series) data, researchers have dealt with this issue by assuming a specific form of factor-biased technological change (such as a log-linear time trend) (Antras, 2004; Herrendorf et al., 2015, among others). On the other hand, one can bring more reasonable instruments when focusing on establishment or firm-level variations. For example, Raval (2019) uses a shift-share variable that captures local impacts of national changes in non-manufacturing industries (i.e., Bartik (1991) instrument) as instruments for shifts in labor market conditions. However, it does not give the aggregate elasticity of substitution as Oberfield and Raval (2021) has shown.

In this section, we first estimate firm-level elasticity of substitution (σ_e and σ_s) and then aggregate them into the aggregate elasticity of substitution using the model and the data on the distribution of factor income shares (i.e., using Proposition 2).

4.1 Data

We use the manufacturing sector data of the Korea Economic Census 2015. It surveys all establishments with more than one employee as of December 31, 2015. We exclude branches, sole proprietorships, governmental and non-profit establishments as they do not report intangible assets. We use annual payroll for wL_i, equipment capital (machinery and transportation equipment) for K_i and software assets for S_i. Lastly, we winsorize factor shares at 1 and 99 percentiles. For comparison, the total payroll and assets in the Census data cover 76 and 54 percent of total compensation of employees and net capital stock in the manufacturing industry in the National Accounts.

We use wage differences across local areas as our main explanatory variable, and we obtain local wages in manufacturing sector from the Regional Employment Survey 2015. The Regional Employment Survey is the household-level survey that reports salary, demographic information, educational attainment, and experience. To control for the skill heterogeneity across regions, we estimate a residual wage for each person controlling for education, experience, and demographics. We then collapse this residual by each local area to get the regional variation in the labor cost.[4]

Finally, we compute capital income by multiplying the rate of return on equipment and software by the stock of equipment and software, respectively. We impute the rate of return on different types of capital from the net capital stock and investment data in the National Accounts (Section 2). To be specific, we assume no-arbitrage conditions across asset types, which gives

$$R_t^j = (1 + r_t)p_{t-1}^j - (1 - \delta_t^j)p_t^j,$$

where r_t is the net rate of return, p_t^j is the price of capital good j relative to consumption good, and δ_t^j is the depreciation rate of the capital good j. Using the data on the net stock of capital K_t^j and investment (I_t), we infer the depreciation rate from

$$\delta_t^j = 1 - \frac{K_t - I_t}{K_{t-1} \times p_t^j / p_{t-1}^j}.$$

Then the net rate of return can be imputed from

$$1 + r_t = \frac{(1/\mu) - \ell_t + \sum_j (1 - \delta_t^j) p_t^j K_t^j / Y_t}{\sum_j p_{t-1}^j K_t / Y_t},$$

where μ is the aggregate markup, ℓ_t is the aggregate labor share, and Y_t is gross value added.

Figure 3 depicts the relationship between relative factor shares and log wage at the regional level. Each point in the figure represents an administrative region. It clearly shows that the relation between software to labor shares ($\log qS/wL$) and local wage ($\log w_r$) is strictly positive whereas the relation between equipment to labor

[4]The unit of a region in our analysis is Si-Gun-Gu, which is an administrative division of South Korea. For comparison, the average population size of commuting zones in the US is around 443.5 thousands, while the average population size of Si-Gun-Gu is 319.2 thousands.

shares ($\log rK/wL$) has slightly negative trend. The difference in the trends suggests a possibility that the elasticities of substitution between labor and capital would be quite heterogeneous by capital type.

Fig. 3: Relation between local wage and factor shares

4.2 Estimation

Using the data we describe above, we run the following regressions at the firm level.

$$\log \frac{rK_i}{wL_i} = \beta_e \log w_r + \gamma_e X + \epsilon_{e,i}, \tag{29}$$

$$\frac{1}{1-k_i} \log \frac{qS_i}{wL_i + rK_i} = \beta_s \log w_r + \gamma_s X + \epsilon_{s,i}, \tag{30}$$

where w_r is the residual wage described above, and X is a set of control variables including three-digit industry fixed effects, age, and multi-unit status dummies. Note that $\beta_e = \sigma_e - 1$ and $\beta_s = \sigma_s - 1$. The specification implicitly assumes that all firms face same cost of capital but different wages across regions.[5]

When using the regional variations, an endogeneity issue could arise if local wages are correlated with unobserved non-neutral productivity. Following Oberfield and Raval (2021) and Raval (2019), we use Bartik (1991)'s instrument for labor market conditions to alleviate this concern. Given initial industrial composition, Bartik (1991)'s

[5]This is why we focus only on equipment rather than structure and equipment. It is unlikely that firms located in different regions face same price of structure.

instrument measures the change in labor market demand due to the nationwide expansion in industrial employment. To be specific, we compute the instrument by

$$Z_r = \sum_{i \in N_s} \omega_{r,i,0} \log(L_{i,t}/L_{i,0}),$$

where N_s is the set of industries in services sector, $\omega_{r,i,0}$ is the industry i's share of employment in region r at time 0, and $L_{i,t}$ is the nationwide employment of industry i.

Because the instrument covers service industries only, we interpret this as a change in the labor market supply of the manufacturing sector.[6] This implicitly assumes that services and manufacturing sector share the common labor supply pool. For this to be true, the most important industries would be those with more of workers who can switch to jobs in the manufacturing industries more easily. Checking the contribution of each industry by the Rotemberg weights computed following Goldsmith-Pinkham et al. (2020), we found that research & development and business support services account for 93% of overall weights (80% of positive weights, details in Appendix B.3.1). Since workers in those two industries are likely to be able to switch to manufacturing sector more easily than workers in other services industries, we view the assumption of the common labor supply pool as a plausible assumption.

Following Goldsmith-Pinkham et al. (2020), we report estimation results with IV regression with two-stage least square (TSLS) and limited-information maximum likelihood (LIML) with each industry share separately as instrument to check the validity and robustness of the Bartik (1991)'s instrument. In addition, we also add the estimation result with an alternative instrument suggested by Beaudry et al. (2012).

Table 5 reports the estimated elasticity of substitution between labor and equipment (σ_e) and between labor and software (σ_s). We report OLS estimates in columns 1, and IV estimates in columns 3 to 6. The elasticity of substitution between labor and equipment ranges from 0.27 to 0.66, depending on the estimation method. These estimates imply the complementarity between labor and equipment, and are in line

[6] The Bartik instrument herein does not account for the industry linkages. To alleviate this concern, Oberfield and Raval (2021) employs a measure of local amenities based on climate and geography as an alternative instrument. However, we do not use a measure of local amenities here because measures of climate and geography do not make enough variation in a small country like Korea. Oberfield and Raval (2021) found that the Bartik instrument and the measure of local amenities do not make a big difference in the estimation of the substitution elasticity.

with other estimates in previous studies in the US, such as Antras (2004); Herrendorf et al. (2015); Knoblach et al. (2020); Oberfield and Raval (2021); Raval (2019).

	OLS	Bartik	BGS	TSLS	LIML
Equipment (σ_e)	0.661 ***	0.493 ***	0.274	0.409 ***	0.400 ***
	(0.084)	(0.153)	(0.220)	(0.069)	(0.095)
Software (σ_s)	1.124 ***	1.697 ***	2.522 ***	1.526 ***	1.535 ***
	(0.119)	(0.229)	(0.350)	(0.106)	(0.134)

Table 5: Estimates of the Firm-Level Capital-Labor Substitution

Column 1 is OLS estimate. Columns 2 and 3 are IV regression using Bartik (1991)'s instrument and Beaudry et al. (2012)'s instrument, respectively. Columns 4 and 5 are IV regression with two-stage least square (TSLS) and limited-information maximum likelihood (LIML) with each industry share separately as instument, respectively. Standard errors are clustered at the level of 3-digit industry and region. *, **, and *** indicate significance at the 10, 5, and 1 percent level, respectively.

Our novel finding is that the elasticity of substitution between labor and software (σ_s) is estimated to be greater than one. It implies that software substitutes labor, and software income share would increase within firms in response to software-augmenting technological change. Notably, $\sigma_s > 1$ ensures that the mechanism in Section 3 is consistent with empirical findings we documented in Section 2, and implies that technological change embodied in software may be an important source of the labor share decline.

Table 6: Robustness Checks for Firm-Level Capital-Labor Substitution

	Benchmark	Positive obs.	Alt. order	Tan/Intan	Alt. wage
Equipment (σ_e)	0.493 ***	0.547 *	0.491 ***	0.654 ***	0.521 ***
	(0.153)	(0.317)	(0.153)	(0.162)	(0.144)
Software (σ_s)	1.697 ***	1.471 ***	1.155 ***	2.815 ***	1.659 ***
	(0.229)	(0.417)	(0.197)	(0.434)	(0.217)

All columns are estimate from IV regression using Bartik (1991)'s instrument. The column 1 is benchmark estimate, and the column 2 is estimates using data with strictly positive K_i^s only. The column 3 is estimation under alternative CES ordering of the production function (31). The column 4 is estimation with total tangible and intangible capital . The column 5 uses the regional average of log hourly wage for the explanatory variable. Standard errors are clustered by three digit industry and local area. *, **, and *** indicate significance at 10%, 5%, and 1%, respectively.

We next check whether the above-unity elasticity of substitution between labor and software is robust. In the Economic Census, many firms report that they do not hold software assets ($S_i = 0$), which might be a measurement error. We first restrict the sample to firms with software assets greater than zero. Next, we check if the results are sensitive to a specific functional form of the firm-level production function (5). More specifically, we consider the alternative ordering of nested CES structure:

$$Y_i = \left[\left(\alpha_i^L (A_L L_i)^{\frac{\sigma_s - 1}{\sigma_s}} + \alpha_i^S (A_S S_i)^{\frac{\sigma_s - 1}{\sigma_s}} \right)^{\frac{\sigma_s(\sigma_e - 1)}{(\sigma_s - 1)\sigma_e}} + \alpha_i^K (A_K K_i)^{\frac{\sigma_e - 1}{\sigma_e}} \right]^{\frac{\sigma_e}{\sigma_e - 1}}, \tag{31}$$

and estimate σ_e and σ_s under this alternative specification. Lastly, we estimate equation (29) and (30) with firm-level wage from the Economic Census (alternative wage 1) and the regional average of the log hourly wage (alternative wage 2) as the explanatory variable. Results are in Table 6, which confirms that equipment complements labor ($\sigma_e < 1$) and software substitutes labor ($\sigma_s > 1$).

4.3 Aggregate Elasticities

We now calculate the aggregate response of factor income shares to the changes in factor prices or factor-biased technological change. Specifically, we apply the model in Section 3 to obtain the aggregate elasticities $\bar{\sigma}_e$ and $\bar{\sigma}_s$ in Proposition 2. The aggregation requires the estimation of ξ's and ζ's and the demand elasticity ϵ. We use the Economic Census to measure the distributional moments, summarized in Table 7. The weights on the between-firm reallocation term (χ, ξ^q, ξ^w, and ξ^r) range from around 0.17 to 0.19. Since the cost to revenue ratio is $(\epsilon - 1)/\epsilon$ in our model, we use the average operational cost to revenue ratio in the Economic Census to get ϵ. The estimated ϵ is about 3.6 in our data (Table 7).

χ	ζ^w	ζ^r	ξ^q	ξ^w	ξ^r	ϵ
0.1337	-0.0028	0.0026	0.1705	0.1986	-0.4286	3.5711

Table 7: Distributional Moments and ϵ

The distributional moments are computed from k_i, ℓ_i, s_i, κ_i, θ_i, and ω_i which we obtain from the Economic Census. Note that $\chi := \frac{\sum_i (k_i - k)^2 \theta_i}{k(1-k)}$, $\xi^w := -\frac{\sum_i \omega_i (s_i - s)(\ell_i - \ell)}{s\ell}$, $\xi^r := -\frac{\sum_i \omega_i (s_i - s)(\kappa_i - \kappa)}{s\kappa}$, $\xi^q := \frac{\sum_i \omega_i (s_i - s)^2}{s(1-s)}$, $\zeta^w := \frac{\sum_i (k_i - k)(1-k_i)\theta_i s_i}{\sum_i (k_i - k)(1-k_i)\theta_i}$, and $\zeta^r := \frac{\sum_i (k_i - k) k_i \theta_i s_i}{\sum_i (k_i - k) k_i \theta_i}$.

Applying the distributional moments to equations (24) to (28) in Proposition 2, we obtain the aggregate elasticities in Table 8. The aggregate elasticity of substitution between labor and equipment lies below one ($\bar{\sigma}_e < 1$). Since our estimates are $\sigma_e <$

$\sigma_s < \epsilon$, the aggregate elasticity of substitution between labor and equipment has to be greater than the firm-level elasticity. Moreover, given $\sigma_s > 1$ and $\epsilon > 1$, the aggregate elasticity might exceed one if the dispersion in the ratio of capital income to labor income (k_i) is large enough. However, the dispersion χ implied by the Economic Census is such that the aggregate elasticity is still below one, consistent with Oberfield and Raval (2021) and others.

	Traditional Capital		Software		
	$\bar{\sigma}_e^w$	$\bar{\sigma}_e^r$	$\bar{\sigma}_s^q$	$\bar{\sigma}_s^w$	$\bar{\sigma}_s^r$
Aggregate Elasticity	0.9053	0.9039	2.0166	2.0691	0.8938

Table 8: Aggregate Elasticities of Substitution

The aggregate elasticities are computed using Proposition 2, where σ_e and σ_s are the IV estimates in Table 5, and χ, ξ, and ζ's are from Table 7.

The results are different for software. Given $1 < \sigma_s < \epsilon$, the aggregate elasticity of substitution between labor and software in response to software price ($\bar{\sigma}_s^q$) has to be always greater than one. Suppose that the price of software dropped one percent exogenously. Since our benchmark σ_s is 1.57, each establishment will substitute toward software so as to increase software income by 57 percent more than non-software income. In addition, this substitution grows larger those firms using software more intensively than others. This between-firm reallocation would increase the software income share by another 38 percent, so the aggregate software income will be 95 percent larger compared to the non-software factor income.

What is interesting is that $\xi^r < 0$, which follows from the positive covariance between equipment share (κ_i) and software share (s_i). Suppose a capital-embodied technological change that lowers r. Because $\sigma_s > 1$, within-firm adjustment implies a reduction in software share. At the same time, firms with higher κ_i will become more productive, and hence increase their share in the value-added (ω_i). Because the covariance between s_i and κ_i is positive, these firms generally have larger s_i's, and hence this reallocation will counteract the within-firm adjustment. The magnitude of this reversal depends on the size of the covariance and the demand elasticity. In this case, the reversal is strong enough that $\bar{\sigma}_s^r$ is slightly less than 1.

5 Implications on the Aggregate Labor Share

To put the estimated elasticities of substitution into context, we quantify the impact of software-embodied technological change on the aggregate labor income share. Researchers have used observed decline in the price of investment relative to the price of consumption to measure the observed technological change specific to capital goods (e.g., Cummins and Violante, 2002). Following the literature, we use changes in the inverse of the price of software and equipment investment relative to consumption to measure software or equipment-specific technological change.

One problem with using the Korean National Accounts is that its implied price index for software investment is not likely to capture software-specific technological progress well. Since 1994, a change in the price index of software investment in the Korean National Accounts comes from the change in the producer price index, which does not capture the quality improvements compared to the hedonic method. Therefore, we adjust the price index following Parker and Grimm (2000), which the Bureau of Economic Analysis does when constructing the price index of software.[7]

We then calculate how much of the labor share decline could be explained by the observed technological change specific to software and/or equipment. From Proposition 2, it is straightforward to see that the impact of investment-specific technological changes ($d\ln 1/q$ or $d\ln 1/r$) on the aggregate labor share is given by

$$\begin{aligned} dLS/LS = &- s(\bar{\sigma}_s^q - 1)d\ln 1/q \\ &- k((\bar{\sigma}_e^r - 1) - s(\bar{\sigma}_s^r - 1))d\ln 1/r. \end{aligned} \tag{32}$$

Given the aggregate elasticities in Table 8, equation (32) implies that the software-embodied technological change ($d\ln 1/q > 0$) reduces the aggregate labor income share. What is interesting is that the effect of $d\ln 1/r$ has two opposing forces. Facing the equipment-specific technological change ($d\ln 1/r > 0$), the first term ($\bar{\sigma}_e^r - 1$) will raise the labor share since $\bar{\sigma}_e^r < 1$. However, since $\bar{\sigma}_s^r$ is also less than one, the second term $-s(\bar{\sigma}_s^r - 1)$ will reduce the labor share. Again, $\bar{\sigma}_s^r$ is less than one because the reallocation is stronger than the within-firm adjustment, raising the software share ($\xi^r < 0$).

[7] The Bureau of Economic Analysis (BEA) recognizes the bias between the hedonic and the matched model method, and makes the bias adjustment of 3.15 percent per year to the producer price index from the Bureau of Labor Statistics. See Parker and Grimm (2000) for details.

To do the decomposition in equation (32), we compute LS_t, k_t, and s_t from the National Accounts.[8] To be specific, we impute the rate of return on capital as described in Section 4.1 and then compute k_t and s_t by multiplying the rate of return to the net capital stock by asset type. Recall that the rate of return satisfies

$$R_t^j = (1 + r_t)p_{t-1}^j - (1 - \delta_t^j)p_t^j,$$

where r_t is net rate of return, p_t^j is the price of capital type j, and δ_t^j is the depreciation rate. Also the observed software- and equipment-specific technological changes are $\ln 1/q = \ln 1/R_t^s$ and $\ln 1/r = \ln 1/R_t^e$, respectively. Note that trends in R_t^j should closely follow p_t^j, especially when r and δ^j's are stable. In the data, the net rate of return r_t shows a declining trend as well, and hence the gross rate of return R_t^j decreases faster than p_t^j.

Even when we assume that the micro elasticity is time-invariant, the aggregate elasticities $\bar{\sigma}$ may vary across time because of the distributional parameters. A problem is that information on the software usages is included in the Census only in 2015. Therefore, we check the pattern of distributional parameters in the frm-level accounting data (KISdata), and then impute the changes in the distributional parameters according to:

$$d_t = d_{2015}^{census} - d_{2015}^{KISdata} + d_t^{KISdata},$$

where d_t is one of the distributional parameters (χ, ζ^w, ζ^r, ξ^q, ξ^w, and ξ^r). It turns out that the consideration of time-varying pattern of distributional parameters does not affect the decomposition result much because they do not have a clear time trend (Details are in Appendix B).

Table 9 depicts how much the software-specific technological change affected the aggregate labor income share between 1990 and 2019. Since 1990, the aggregate labor share has declined by 4.9 percentage points. The observed software-embodied technological change accounts for a 2.5 p.p. reduction in the aggregate labor share, which is 51.2 percent of the overall decline. Of this, 33.4 percent point is due to within-firm substitution and the remaining 17.8 percent point is due to between-firm reallocation.

[8] Although the elasticity estimation uses manufacturing data, the decomposition is based on the aggregate labor share. This is because the National Accounts data do not report k_t, s_t, q_t, and s_t by industry in Korea. Using the US data, Aum and Shin (2020) showed that the correlation between software intensity and the labor share decline is stronger in the services industry than in the manufacturing industry.

At the same time, the equipment-embodied technological change accounts for a 1.1 p.p. increase in the labor share. In total, unlike in Oberfield and Raval (2021), overall capital-embodied technological change has reduced the labor share when software and equipment are considered separately.

	Labor Share	Software ($\Delta \ln 1/q$)			Equipment ($\Delta \ln 1/r$)
		Total	Within	Reallocation	
Changes	-0.049	-0.025	-0.016	-0.009	0.011
(% of total)		(51.2)	(33.4)	(17.8)	(-22.0)

Table 9: Effects of Capital-Embodied Technological Change on the Labor Share

We adjust for the proprietors' labor income in the aggregate labor share following Gollin (2002). The decomposition is for the periods between 1990 and 2019. Percent explained of the overall labor share decline in parentheses.

In equation (32), software-embodied technology affects the labor share only through capital-labor substitution. This is because we assume a constant demand elasticity (ϵ) and hence technological changes do not affect the markup. Our empirical finding in Table 3 in Section 2, however, indicates that software-embodied technology can also affect the markup. Although not explicitly analyzed in the model, we took a naive look at how the impact of software on markups will in turn affect the labor share.

More specifically, for every year, we compute an implied increase in the log of markups due to software by multiplying the estimated coefficient in Table 3 ($\hat{\beta} = 0.34$) by the annual change in aggregate software capital to value-added in the data. That is, the implied changes in the markup would be $\Delta \ln \mu \approx \hat{\beta} \times \Delta s_t$, which implies a further decline in the aggregate labor share by $LS_t \times \Delta \ln \mu$ due to software. The results are summarized in Table 10. Between 1990 and 2019, this would imply -1.3 p.p. further decline in the labor share, explaining additional 25.7 percent of the overall decline in the aggregate labor share.

The decomposition results imply that the labor share would have declined more slowly if not for the software-embodied technological change. Figure 4 depicts how the aggregate labor share would have evolved without software-specific technological change. We confirm that the counterfactual labor share ($LS \times (1 + \sum[s(\bar{\sigma}_s^q - 1) d \ln 1/q + d \ln \mu])$, red line) declines more slowly than the actual labor share (blue line), with an estimated trend (-0.17) about 60% of the trend in the data (-0.28).

	Labor Share	$\Delta \ln 1/q$	$\Delta \ln \mu$	Total
Changes	-0.049	-0.025	-0.013	-0.038
(% of total)		(51.2)	(25.7)	(76.9)

Table 10: The Implied Effects from the Changes in Markup

We adjust for the proprietors' labor income in the aggregate labor share following Gollin (2002). The decomposition is for the periods between 1990 and 2019. Percent explained of the overall labor share decline in parentheses.

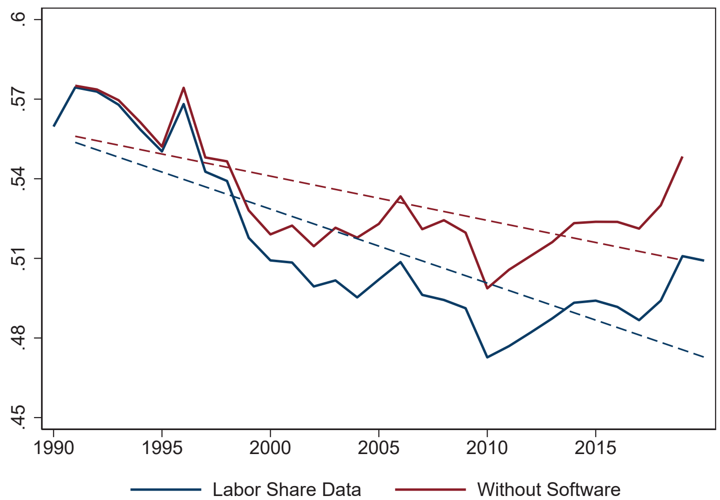

Fig. 4: Comparison between the Actual and the Counterfactual Labor Share

The labor share without software is $LS \times (1 + \sum[s(\bar{\sigma}_s^q - 1)d \ln 1/q + d \ln \mu])$.

Alternative Demand Elasticities The aggregate elasticities ($\bar{\sigma}_e$ and $\bar{\sigma}_s$) used for the decomposition in equation (32) come from Table 8, which uses the demand elasticity ϵ we estimated from Economic Census 2015. To see how different ways of estimating ϵ would affect the results, we redo the decomposition exercise with the demand elasticity implied by the harmonic mean of firm-level markups ($\bar{\mu} := \epsilon/(\epsilon - 1)$) estimated from firms' financial information in the KISDATA database in Section 2 and Appendix B. Specifically, we use the harmonic mean of firm-level markups in the manufacturing sector, where the markups are estimated via three methods: accounting profit (μ^{AP}), user cost (μ^{UC}), and production function approach (μ^{PF}).

Figure 5 plots the counterfactual labor shares without software-embodied technological change using alternative markup estimates. The markup level estimated from firm-level data is generally lower than what we obtain from the Economic Census.

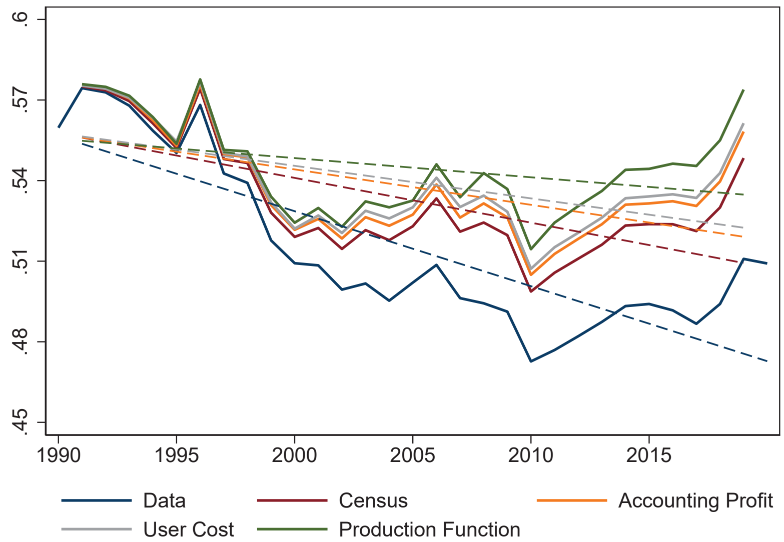

Fig. 5: Actual and Counterfactual Labor Share with Alternative Markups

The labor share without software is $LS \times (1 + \sum[s(\bar{\sigma}_s^q - 1)d\ln 1/q + d\ln\mu])$.

Therefore, the implied demand elasticity is larger, and reallocation becomes more important. As a result, we get much flatter counterfactual labor shares under alternative markups. For example, when the markup estimated from the production function approach is used to compute the demand elasticity, the declining trend in the counterfactual labor share becomes much flatter (-0.07), about one quarter of the actual trend (-0.28). In other words, the importance of software in the decline of the aggregate labor share is understated in our benchmark exercise.

6 Conclusion

In this paper, we establish that the capital-labor substitution elasticity is different across types of capital both qualitatively and quantitatively. In particular, we show that the software-labor substitution elasticity is greater than one but the equipment-labor substitution elasticity is less than one.

The difference is essential in understanding the aggregate labor income share decline. We show that decline in the price of software (or software-embodied technological change) connects the three leading explanations for the labor share decline. First, the above-unitary elasticity of substitution between labor and software is consistent with the argument that the observed technological change embodied in capital lowers the labor share (Karabarbounis and Neiman, 2013). However, it is only software (or intangible) capital that has an elasticity greater than one, consistent with the fact that labor share declines only when accounting for the intangible investment in value-added (Koh et al., 2020). At the micro-level, firms with higher software intensity generally have low labor shares, and the aggregate elasticity implies that firms with low labor shares also grow their size with software-embodied technological change, consistent with the reallocation channel emphasized in the literature (Autor et al., 2020; Kehrig and Vincent, 2021).

Quantitatively, the measured technological change embodied in software explains about half of the labor share decline in Korea between 1990 and 2019 under the most conservative specification. Among them, 33 p.p. is due to within-firm adjustment and the remaining 18 p.p. is due to reallocation from low to high software intensity firms. Depending on the specification, eliminating the software-embodied technological change can wipe out up to 77 percent of the downward trend in the aggregate labor share. Software, not equipment, is the key to understanding the decline of the labor share.

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Appendix A Proofs

Proof of Proposition 1 From equation (9) and (10), we have the relative price between wage and the price of traditional capital as:

$$\frac{w}{r} = \frac{\alpha_i^L}{\alpha_i^K}\left(\frac{A_L}{A_K}\right)^{\frac{\sigma_e-1}{\sigma_e}}\left(\frac{K_i}{L_i}\right)^{\frac{1}{\sigma_e}}.$$

Taking logs, we have

$$(\sigma_e - 1)\ln\frac{w}{r} = \sigma_e \ln\frac{\alpha_i^L}{\alpha_i^K} + (\sigma_e - 1)\ln\frac{A_L}{A_K} + \ln\frac{rK_i}{wL_i}, \tag{A.1}$$

Differentiating equation (A.1) with fixing A_L/A_K, and using the definition of k_i, we get equation (15), which is the first part of Proposition 1.

Similarly, from equation (6), (9), (10), and (13), we have $\phi_i = p_i(Y_i/X_i)^{1/\sigma_s}$. And divided it by (12), we have

$$\frac{\phi_i}{q} = \frac{1}{\alpha_i^S}\left(\frac{1}{A_S}\right)^{\frac{\sigma_s-1}{\sigma_s}}\left(\frac{S_i}{X_i}\right)^{\frac{1}{\sigma_s}}.$$

Taking logs, we have

$$(\sigma_s - 1)\ln\frac{\phi_i}{q} = \sigma_s \ln\frac{1}{\alpha_i^S} + (\sigma_s - 1)\ln\frac{1}{A_S} + \ln\frac{qS_i}{\phi_i X_i} \tag{A.2}$$

Differentiating equation (A.2) with fixing A_S, we get

$$\sigma_s - 1 = \frac{d\ln\frac{qS_i}{\phi_i X_i}}{d\ln\frac{\phi_i}{q}}. \tag{A.3}$$

From the definition of ϕ_i (13),

$$d\ln\phi_i = \frac{1}{1+\left(\frac{\alpha_i^K}{\alpha_i^L}\right)^{\sigma}_e\left(\frac{rA_L}{wA_K}\right)^{1-\sigma_e}} d\ln w + \frac{1}{1+\left(\frac{\alpha_i^L}{\alpha_i^K}\right)^{\sigma}_e\left(\frac{wA_K}{rA_L}\right)^{1-\sigma_e}} d\ln r \tag{A.4}$$

Since $\left(\frac{r\alpha_i^L}{w\alpha_i^K}\right)^{\sigma}_e\left(\frac{A_L}{A_K}\right)^{\sigma_e-1}\frac{w}{r} = \frac{wL_i}{rK_i}$ from equation (9) and (10), by inserting it into equation (A.4) and using the definition of k_i, we have

$$d\ln\phi_i = (1-k_i)d\ln w + k_i d\ln r.$$

Lstly, we have $\phi_i X_i = wL_i + rK_i$ from equation (6), (9), (10), and (13). So equation (A.3) becomes

$$\sigma_s - 1 = \frac{d\ln\frac{qS_i}{wL_i + rK_i}}{(1-k_i)d\ln\frac{w}{q} + k_i d\ln\frac{r}{q}},$$

which is the second part of Proposition 1. ∎

Proof of Corollary 1 Totally differentiating equation (A.1) and setting $d\ln w = d\ln r = 0$, it is straightforward to see the first part of Corollary 1. Also, totally differntiating equation (A.3) and setting $d\ln q = 0$, and following the same steps in the proof of Proposition 2, the second part of Corollary 1 follows. ∎

Proof of Proposition 2 Consider $d\ln w > 0$, $d\ln r = 0$, and $d\ln q = 0$.

Traditional capital-labor substitution From the definition of the aggregate elasticity (24),

$$\bar{\sigma}_e^w - 1 = \frac{d\ln\frac{rK}{wL}}{d\ln w} = \frac{d\ln\frac{k}{1-k}}{d\ln w} = \frac{1}{1-k}\frac{d\ln k}{d\ln w} = \frac{1}{k(1-k)}\frac{dk}{d\ln w} \tag{A.5}$$

From the definition of $k := \sum \theta_i k_i$,

$$dk = \sum_i \theta_i dk_i + \sum_i k_i d\theta_i \tag{A.6}$$

From the equation (15) in Proposition 1, we know that

$$dk_i = (\sigma_e - 1)k_i(1-k_i)d\ln w \tag{A.7}$$

Also from the definition of θ_i,

$$\begin{aligned}\theta_i &= \frac{wL_i + rK_i}{wL + rK} \\ &= \frac{wL_i + rK_i}{wL_i + rK_i + qS_i} \times \frac{wL_i + rK_i + qS_i}{wL + rK + qS} \times \frac{wL + rK + qS}{wL + rK} \\ &= \frac{1-s_i}{1-s}\omega_i.\end{aligned}$$

Since $\omega_i = \frac{p_iY_i}{pY} = \gamma_i^{\epsilon}\left(\frac{p_i}{p}\right)^{1-\epsilon}$,

$$d\theta_i = \theta_i d\ln\theta_i = \theta_i d\ln(1-s_i) - \theta_i d\ln(1-s) + \theta_i(1-\epsilon)(d\ln p_i - d\ln p) \tag{A.8}$$

From equation (16) in Proposition 1, equation (A.8) is

$$d\theta_i = -\theta_i s_i(\sigma_s - 1)(1-k_i)d\ln w - \theta_i d\ln(1-s) + \theta_i(1-\epsilon)(d\ln p_i - d\ln p) \tag{A.9}$$

Following Oberfield and Raval (2021), we will use the fact that $\sum_i k d\theta_i = 0$ and $\sum_i(k_i - k)\theta_i = 0$. Using these facts, equation (A.9), and $d\ln p_i = (1-s_i)(1-k_i)d\ln w$,

$$\sum_i k_i d\theta_i = \sum_i (k_i - k)d\theta_i$$

$$
\begin{aligned}
&= \sum_i (k_i - k)\theta_i \left[(1-\sigma_s)s_i(1-k_i)d\ln w + (1-\epsilon)d\ln p_i\right] \\
&= \sum_i (k_i - k)\theta_i \left[(\epsilon-\sigma_s)(1-k_i)s_i + (1-\epsilon)(1-k_i)\right] d\ln w. \qquad \text{(A.10)}
\end{aligned}
$$

Substituting (A.7) and (A.10) into (A.6),

$$
dk = \sum_i \theta_i(1-k_i)\left[(\sigma_e - 1)k_i + (k_i-k)(\epsilon-\sigma_s)s_i + (k_i-k)(1-\epsilon)\right] d\ln w \quad \text{(A.11)}
$$

From (A.5), (A.11), and $\sum_i (k_i - k)\theta = 0$,

$$
\begin{aligned}
\bar{\sigma}_e^w - 1 &= \frac{\sum_i \theta_i(1-k_i)k_i}{(1-k)k}(\sigma_e - 1) \\
&+ \frac{\sum_i \theta_i(1-k_i)(k_i-k)s_i}{k(1-k)}(\sigma_s - \epsilon) \\
&+ \frac{\sum_i \theta_i(k_i-k)^2}{k(1-k)}(\epsilon - 1) \\
&= (1-\chi)(\sigma_e - 1) + \zeta^w\chi(\sigma_s - \epsilon) + \chi(\epsilon - 1) \\
&= (1-\chi)\sigma_e + \chi\zeta^w\sigma_s + \chi(1-\zeta^w)\epsilon - 1,
\end{aligned}
$$

which is equation (24) in Proposition 2.

Software-labor substitution Similarly, from the definition of $\bar{\sigma}_s^w$,

$$
\bar{\sigma}_s^w - 1 = \frac{d\ln\frac{s}{1-s}}{(1-k)(1-s)d\ln w} = \frac{1}{s(1-s)(1-k)}\frac{ds}{d\ln w} = \frac{1}{s\ell}\frac{ds}{d\ln w} \qquad \text{(A.12)}
$$

Since $s = \sum_i \omega_i s_i$,

$$
ds = \sum_i \omega_i ds_i + \sum_i s_i d\omega_i. \qquad \text{(A.13)}
$$

From the equation (16) in Proposition 1, we see that

$$
ds_i = (\sigma_s - 1)s_i\ell_i d\ln w, \qquad \text{(A.14)}
$$

where we use that $\ell_i = (1-s_i)(1-k_i)$.

We already know that $d\omega_i = \omega_i(1-\epsilon)\left(d\ln p_i - d\ln p\right)$ and $d\ln p_i = \ell_i d\ln w$. So, we have

$$
d\omega_i = \omega_i(1-\epsilon)\left(\ell_i d\ln w - d\ln p\right)
$$

Since $\sum_i s d\omega_i = 0$,

$$
\sum_i s_i d\omega_i = \sum_i (s_i - s)d\omega_i
$$

$$= \sum_i (s_i - s)\omega_i(1-\epsilon)\ell_i d\ln w$$
$$= \sum_i (s_i - s)\omega_i(1-\epsilon)(\ell_i - \ell) d\ln w \tag{A.15}$$

Substituting equation (A.14) and (A.15) into equation (A.13),

$$ds = \sum_i \omega_i \left[(\sigma_s - 1)s_i\ell_i + (s_i - s)(\ell_i - \ell)(1-\epsilon)\right] d\ln w \tag{A.16}$$

Also, note that

$$1 + \frac{\sum_i (s_i - s)(\ell_i - \ell)\omega_i}{s\ell} = \frac{\sum_i \left[(s_i - s)(\ell_i - \ell) + s\ell\right]\omega_i}{s\ell} = \frac{\sum_i s_i\ell_i\omega_i}{s\ell}. \tag{A.17}$$

Substituting equation (A.16) and (A.17) into (A.12),

$$\bar{\sigma}_s^w - 1 = (\sigma_s - 1)(1 - \xi^w) + \xi^w(\epsilon - 1), \tag{A.18}$$

which is equation (27) in Propositoin 2.

Cases with $d\ln r > 0$ are $d\ln q > 0$ are analogous. ∎

Appendix B Data

B.1 KISDATA database

The KISDATA is a database on financial information for firms listed on the Korea Stock Exchange and firms unlisted but required to publish external auditing reports. Criteria for external auditing requirement is as follows. Until 2008, firms whose asset value exceeded 7 billion KRW had to be audited externally. Since 2009 (2014), (i) firms with asset value greater than 10 billion (12 billion) KRW, (ii) asset value greater than 7 billion KRW and liability greater than 7 billion KRW, or (iii) asset value greater than 7 billion KRW and the number of employees more than 300 were subject to external auditing. Among firms included in the KISDATA, we exclude financial firms and quasi-governmental and non-profit firms from the sample. Our dataset covers the years between 2000 and 2018.

B.1.1 Labor Share

Construction of the labor share requires data on labor compensation and value added. We combine employee compensation and benefits in the income statement and the labor cost in the manufacturing cost statement to obtain a firm's total labor compensation. Note that the employee compensation and benefits in the income statement can be understood as labor income accruing to non-production workers, whereas the labor cost in the manufacturing cost statment can be regarded as labor income accruing to production workers.

To compute value added, we add up operational profit, depreciation and amortization, taxes and dues, and labor compensation. The labor share at the firm level is then computed by the labor compensation divided by value added. In the regression analysis, we keep the observations with the labor share between zero and one.

B.1.2 Software and Equipment capital

We use the variable "intangible asset - software" in firms' balance sheet to measure their software asset. according to Korean Generally Accepted Accounting Principles (K-GAAP), a firm classifies its software purchases from outside as software assets (in intangible assets). A firm may have software developed in-house as intangible asset, but this component is not included in our analysis as it's included in the research and development and not separately reported. Also, our measure of equipment capital is sum of machinery and transportation equipment, reported in the balance sheet data. We divide software asset and equipment asset by value added to measure software intensity and equipment intensity, respectively. For the regression analysis, we winsorize software and equipment intensity at 1-99 percentile by year.

B.1.3 Markup

Detailed procedure for the construction of markup mostly follows Baqaee and Farhi (2019).

Accounting Profit For the accounting profit approach, we use operating income to measure profits and use the expression

$$profit = \left(1 - \frac{1}{\mu^{AP}}\right) sales, \tag{B.1}$$

to get μ^{AP} for each firm in each year.

User Cost The user cost approach computes income to the capital with multiple of the user cost of capital and capital stock. For this approach, we assume that operating surplus is

$$OS = RK + \left(1 - \frac{1}{\mu^{OC}}\right) sales, \tag{B.2}$$

where OS is the operating income (with depreciation), R is the user-cost of capital, and K is the quantity of capital. We use the sum of sales net of cost of goods sold and depreciation to get OS, and the sum of tangible and intangible assets to measure K.

The user cost of capital is given by

$$R_{i,t} = (1 + r_t) - (1 - \delta_{i,t}) E_t p^k_{i,t+1} / p^k_{i,t}, \tag{B.3}$$

where the r is the average real rate of the commercial paper, $\delta_{i,t}$ is the industry-level depreciation rate implied in the National Account, and $E_t p^k_{i,t+1} / p^k_{i,t}$ is three-year moving average of the changes in the relative price of capital to consumption by industry. Then we back out μ^{UC} from equation (B.2) and (B.3).

Production Function For the production function approach we estimate elasticity of output with respect to variable inputs following Baqaee and Farhi (2019) and De Loecker et al. (2020).

To estimate elasticity, we need outcome variable (log sales), free variable (log cost of goods sold), state variable (log capital stock), and proxy variable (log investment). We deflate sales and cost of goods sold with gross value added deflator by industry and capital expenditure with gross fixed capital formation deflator by industry. To compute capital stock, we apply the perpetual inventory method (PIM) with the initial level of tangible and intangible capital and capital expenditure. We also control sales share in one-digit and two-digit industries in the estimation. In the estimation, we exclude samples with the cost of goods sold to sales ratio or selling, the general and administrative expense to sales ratio in the top and bottom 2.5% by year. We also exclude agricuture as well as finance and insurance industry.

The elasticity is estimated using Olley and Pakes (1996) with three-year rolling windows by one-digit industry, and μ^{PF} is then

$$\mu^{PF} = \frac{\partial \log F / \partial \log X}{X/Y}, \tag{B.4}$$

where F is production function, X is variable input (cost of goods sold), and Y is sales turnover.

Lastly, when we do the regression analysis, we winsorize log of markups at 1% and 99% by year.

B.1.4 TFP

We estimate firm-level TFPs from the estimation of production fuction we did for the construction of markups. That is, we estimate the production function using Olley and Pakes (1996)'s method with three-year rolling windows by one-digit industry, and compute log of the total factor productivity accordingly. We also winsorize log of the total factor productivity at 1% and 99% by year.

B.1.5 Distributional Parameters

To check time variations in the distributional parameters, we compute the distributional parameters (χ, ζ^r, ζ^w, ξ^r, ξ^w, and ξ^q) using the KISDATA dataset. Because the firms listed on the Korea Stock Exchange have adopted International Financial Reporting Standards (IFRS) since 2011, there are some firms who did not report software asset since 2011. In this case, we impute $qS_{i,t}$ with $qS_{i,t-1}$. Computing the distributional parameters, we keep the observations with $rK_i \geq 0$, $qS_i \geq 0$, and $wL_i \geq 0$, where wL_i is the total labor compensation in firm i from the firm's income statement and manufacturing cost statement. We get rK_i by the multiplication of the rate of return on equipment and the equipment capital stock, and qS_i by the multiplication of the rate of return on software and the software capital stock. Here, the rate of return on capital by capital type is imputed from the aggregate National Account data. To be specific, we impute the rate of return on capital assuming no arbitrage condition given as following.

$$R_t^j = (1 + r_t)p_{t-1}^j - (1 - \delta_t^j)p_t^j,$$

where r_t is the net rate of return, p_t^j is the price of capital good j relative to consumption good, and δ_t^j is the depreciation rate of the capital good j. Then the net rate of return can be imputed from

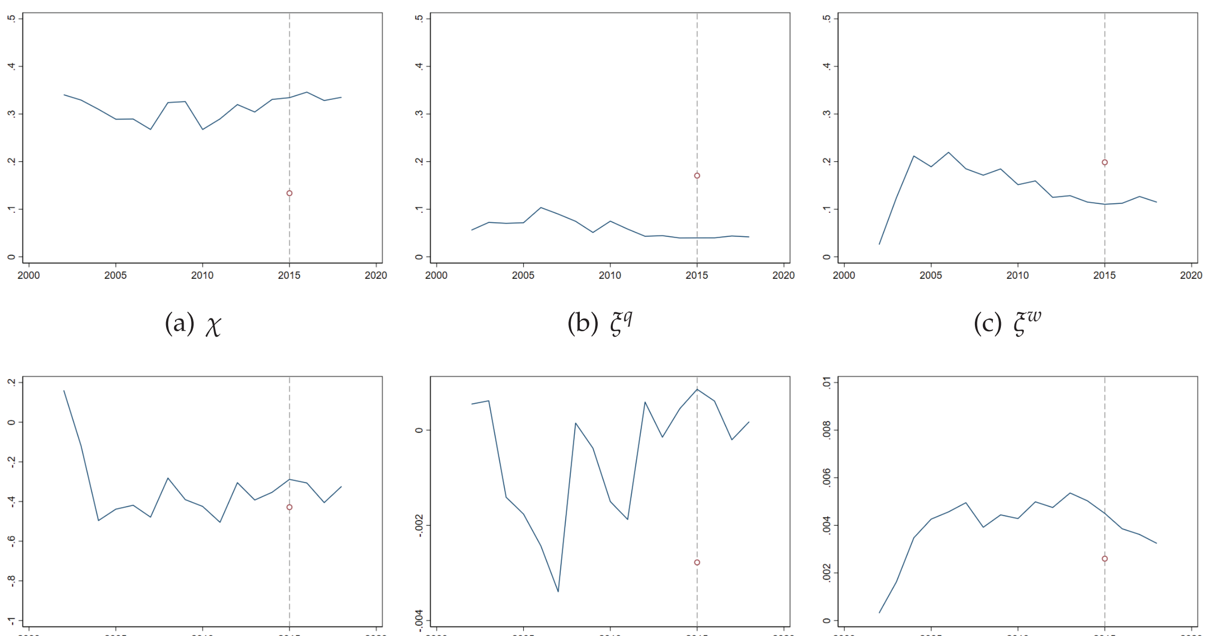
$$1 + r_t = \frac{(1/\mu) - \ell_t + \sum_j (1 - \delta_t^j)p_t^j K_t^j / Y_t}{\sum_j p_{t-1}^j K_t / Y_t},$$

where μ is the aggregate markup, ℓ_t is the aggregate labor share, and Y_t is gross value added.

Then the distributional parameters are compauted by $\chi := \frac{\sum_i (k_i - k)^2 \theta_i}{k(1-k)}$, $\xi^w := -\frac{\sum_i \omega_i (s_i - s)(\ell_i - \ell)}{s\ell}$, $\xi^r := -\frac{\sum_i \omega_i (s_i - s)(\kappa_i - \kappa)}{s\kappa}$, $\xi^q := \frac{\sum_i \omega_i (s_i - s)^2}{s(1-s)}$, $\zeta^w := \frac{\sum_i (k_i - k)(1-k_i)\theta_i s_i}{\sum_i (k_i - k)(1-k_i)\theta_i}$, and $\zeta^r := \frac{\sum_i (k_i - k) k_i \theta_i s_i}{\sum_i (k_i - k) k_i \theta_i}$ as described in Section 3.

Figure B.1 depicts the distributional parameters computed from the KISDATA database. Two things are noteworthy. First, none of the distributional parameters have a clear time trend although there are some fluctuations over time. Second, they are not qualitatively different from the values computed from the Census data. Quantitatively, the parameter related to the dispersion of equipment (χ) is higher in the KISDATA, and the parameter related to the dispersion of software (ξ^q) is higher in the Census data.

Fig. B.1: Distributional Parameters computed from the KISDATA database



(a) χ (b) ξ^q (c) ξ^w

(d) ξ^r (e) ζ^w (f) ζ^r

The circle represents the distributional parameters computed from the Census data.

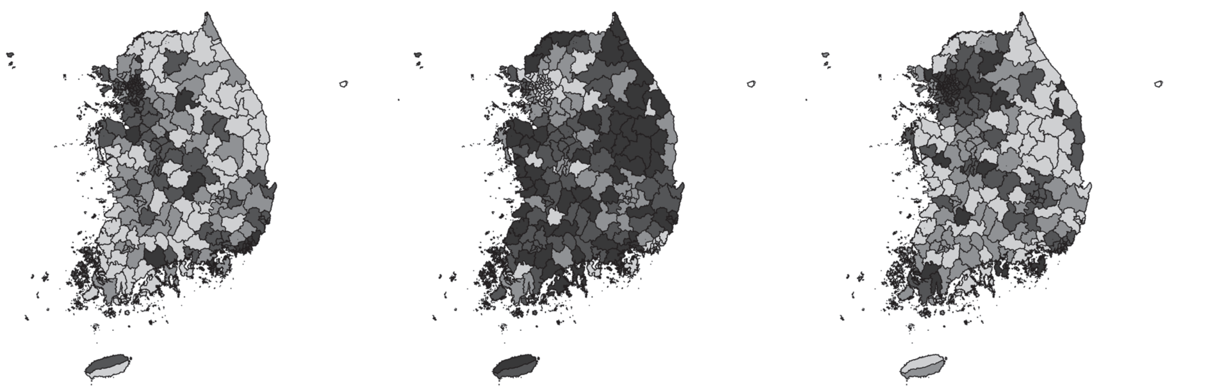
B.2 Census

Our primary data source for the estimation of the elasticity of substitution between labor and capital is the manufacturing sector data of the Korea Economic Census 2015. It surveys all establishments with more than one employee as of December 31, 2015. We exclude branches, sole proprietorships, governmental and non-profit establishments as they do not report intangible assets. We use annual payroll for wL_i, equipment capital (machinery and transportation equipment) for K_i and software assets for S_i. We drop all the establishments that did not report whether they have intangibles. In case an establishment explicitly reports that it does not hold intangibles, we assign zero values to S_i. To compute the factor income shares, we use the rate of return on equipment and software (r and q) imputed from the National Accounts. Lastly, we winsorize factor shares at 1 and 99 percentiles.

B.2.1 Distribution of factor income shares

Census data includes information on the location of establishments and the unit of a region in our analysis is Si-Gun-Gu, an administrative division of South korea, comparable with commuting zones in the US in terms of the average population size. Figure B.2 shows the regional distribution of software (s_i), equipment (k_i), and labor income shares (ℓ_i).

Fig. B.2: Factor income shares by region

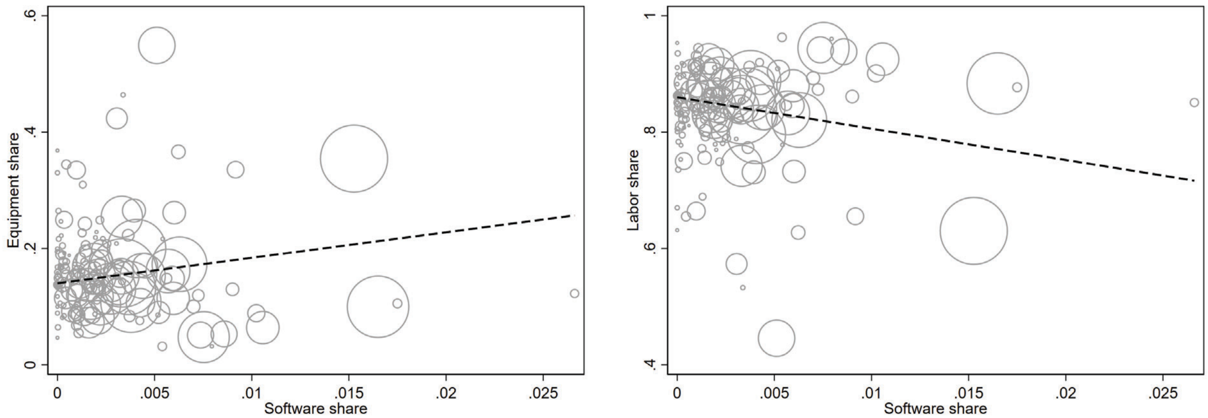


(a) Software (s_i) (b) Equipment (k_i) (c) Labor (ℓ_i)

The regions are classified according to the quantiles of the distribution of factor income shares.

Note that the covariance between software shares (s_i) and equipment shares (k_i) is positive and the covariance between software shares (s_i) and labor income shares (ℓ_i) is negative as indicated by the signs of ξ^r and ξ^w. These relationships are more clearly seen in Figure B.3 that relates software share and equipment or labor income shares.

Fig. B.3: Relationship between software income share and labor or equipment income share



(a) Software vs Equipment

(b) Software vs Labor

B.2.2 Simulations on the changes in distributional parameters

To get an idea on how capital-embodied technological changes whould have affected the distributional patterns of factor income shares, we simulate changes in factor income shares responding to the observed decline in the price of equipment and software with the model. More specifically, from proposition 1, 2, and definition 1:

$$dk_i = k_i(1-k_i)(\sigma_e - 1)(d\ln w - d\ln r)$$

$$ds_i = s_i(1-s_i)(\sigma_s - 1)((1-k_i)d\ln w + k_i d\ln r - d\ln q)$$

$$dk = k(1-k)\left[(\bar{\sigma}_e^w - 1)d\ln w - (\bar{\sigma}_e^r - 1)d\ln r\right]$$

$$ds = s(1-s)\left[(\bar{\sigma}_s^w - 1)(1-k)d\ln w + (\bar{\sigma}_s^r - 1)kd\ln r - (\bar{\sigma}_s^q - 1)d\ln q\right]$$

$$d\omega_i = \omega_i(1-\epsilon)(\ell_i d\ln w + \kappa_i d\ln r + s_i d\ln q - d\ln p)$$

$$d\ln p = \sum_i \omega_i d\ln p_i = \sum_i \omega_i(\ell_i d\ln w + \kappa_i d\ln r + s_i d\ln q)$$

$$d\theta_i = \theta_i\left[-\frac{1}{1-s_i}ds_i + \frac{1}{1-s}ds + \frac{1}{\omega_i}d\omega_i\right]$$

Using these relations, we can obtain the distribution of factor income shares implied by the capital embodied technological changes in the data ($-d\ln q$ and $-d\ln r$).

B.3 Regional Employment Survey

We use wage differences across local areas as our main explanatory variable in the estimation of the elasticity of substitution, We obtain local wages in manufacturing sector from the Regional Employment Survey 2015. The Regional Employment Survey is the household-level survey that reports salary, demographic information, educational attainment, and experience. To control for the skill heterogeneity across regions, we estimate a residual wage for each person controlling for education, experience, and demographics. We then collapse this residual by each local area to get the regional variation in the labor cost.

B.3.1 Bartik instrument

To alleviate a concern on unobserved non-neutral productivity correlated with local wages, we use Bartik (1991)'s instrument measures. Given initial industrial composition, Bartik (1991)'s instrument measures the change in labor market demand due to the nationwide expansion in industrial employment. Specifically, we compute the instrument by

$$Z_r = \sum_{i \in N_s} \omega_{r,i,0} \log(L_{i,t}/L_{i,0}),$$

where N_s is the set of industries in services sector, $\omega_{r,i,0}$ is the industry i's share of employment in region r at time 0, and $L_{i,t}$ is the nationwide employment of industry i. Because the instrument covers service industries only, we interpret this as a change in the labor market supply of the manufacturing sector.

Goldsmith-Pinkham et al. (2020) documented that the bartik estimator ($\hat{\beta}_{bartik}$) can be decomposed into the weighted sum of the just identified estimators with one industry's regional share as an instrument ($\hat{\beta}_{bartik} = \sum_i \hat{\alpha}_i \hat{\beta}_i$), where the weights ($\alpha_i$'s) are the Rotemberg weight. Table B.1 shows summary of the Rotemberg weights. It shows that two industries with the largest weights account for over 90 percent of the overall weights and 64 (=0.93/1.45) percent of the positive weight in the estimator. These are research & development and business support services. ince workers in those two industries are likely to be able to switch to manufacturing sector more easily than workers in other services industries, we view this as assuring result regarding the assumption of the common labor supply pool.

Panel B shows that the national growth rates (g_i) are quite related to the weights while the the variation in the industry shares across locations ($var(\omega_i)$) are only weakly correlated.

Table B.1: Summary of the Rotemberg Weights

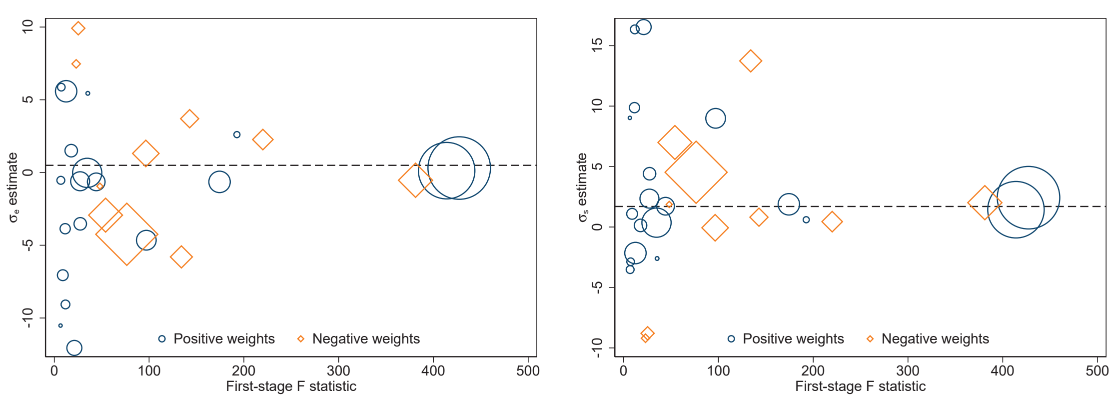
	sum	mean	share
Panel A. Negative and positive weights			
Negative	-0.446	-0.041	0.236
Positive	1.446	0.063	0.764

	$\hat{\alpha}_i$	g_i	$\hat{F}_i$	$var(\omega_i)$
Panel B. Correlations				
$\hat{\alpha}_i$	1.000			
g_i	0.248	1.000		
$\hat{F}_i$	0.589	0.114	1.000	
$var(\omega_i)$	-0.034	-0.104	0.098	1.000

	$\hat{\alpha}_i$	g_i	$\hat{\sigma}_e$	$\hat{\sigma}_s$	$\sum_{k=1}^{i} \hat{\alpha}_k$
Panel C. Top Rotemberg weight industries					
Research & development	0.513	0.316	0.309	2.425	0.513
Business support services	0.418	0.182	0.128	1.431	0.932
Warehousing and transportation	0.112	0.165	-0.025	0.360	1.043
Retail trade (except motor)	0.060	-0.023	-0.644	1.878	1.104

The panel B reports correlations between the weights ($\hat{\alpha}_i$), the nationwide growth of industry i (g_i), the first-stage F-statistic of the industry share ($\hat{F}_i$), and the variation in the industry shares across locations ($var(\omega_i)$).

Fig. B.4: Heterogeneity of the elasticities



(a) Equipment ($\hat{\sigma}_e$) (b) Software ($\hat{\sigma}_s$)

The figure plots the relationship between each instruments' $\hat{\sigma}_k$, first-stage F-statistics, and the Rotemberg weights. Each point is a separate instrument's estimates. The estimated ($\hat{\sigma}_k$) for each instrument is on the y-axis and the estimated first-stage F-statistic is on the x-axis. The size of the points are scaled by the magnitude of the Rotemberg weights, with the circles denoting positive weights and the diamonds denoting negative weights. The dashed line is at the value of the Bartik estimator on the elasticity of substitution. The figure excludes instruments with first-stage F-statistics below 5.

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