Analysis of the CME CF Bitcoin Reference Rate and CME CF Bitcoin Real Time Index

Andrew Paine and William J. Knottenbelt

Imperial College Centre for Cryptocurrency Research and Engineering

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1 Introduction

The goal of this report is to analyse the methodology for calculating a pair of indices for the US Dollar price of bitcoin – the CME[1] CF[2] Bitcoin Reference Rate (BRR) and the CME CF Bitcoin Real Time Index (BRTI). The methods for calculating each rate will be assessed with respect to their ability to satisfy a set of desirable characteristics for indices, as follows:

- Relevance – the index should reflect the supply and demand and the resulting true value of the underlying asset as accurately as possible.
- Timeliness – the index should refer to a specific point in time or a time interval as short as possible and shall be available as soon as possible after that point or interval.
- Manipulation Resistance – it shall be as expensive as possible to move the index away from the true value of the underlying asset.
- Martingale Property – it shall be impossible to game a derivatives market by predicting patterns in the price behaviour of the index.
- Verifiability – the methodology by which the index is calculated shall be transparent and its input data shall be readily available such that calculation results can be independently verified.
- Replicability – investors shall be able to reproduce the index with minimal tracking error by trading in the spot market of the underlying asset.
- Stability – the index shall not exhibit price fluctuations other than those caused by the actual changes in the true value of the underlying asset and shall not be susceptible to outliers and data quality issues.
- Parsimony – the methodology of the index shall utilize as few arbitrary parameters as possible.

The graphs used in this report are calculated during "distressed periods" that correspond to periods of high volatility for each index respectively. **Note that the "Constituent Exchanges" (CEs) used in this report are selected for illustrative purposes only and do not necessarily reflect the exchanges that are included in the actual index, which may vary over time.**

1.1 Bitcoin

Bitcoin is a protocol and network for the exchange of value without a central authority. Bitcoins (BTC) are a digital currency that can be transacted nearly instantaneously on the Bitcoin network. These transactions are all logged on a ledger known as the blockchain that is maintained by a large number of computers (nodes) that come to an agreement on the state of the transaction ledger. All transactions are cryptographically signed to ensure their authenticity.

[1] CME Group, operator of the CME, CBOT, NYMEX and COMEX exchanges and supplier of a wide range of global benchmark products across all major asset classes

[2] Crypto Facilities Ltd, a London-based financial services firm that provides risk management and trading solutions around digital assets such as bitcoin

1.2 Mining and Exchanges

New bitcoins may only be obtained through an automatic process known as mining – equivalent to minting new fiat currency. As the network has grown, and mining hardware has evolved, it has become both technically and financially expensive to participate in this mining in a meaningful way; hence most individuals obtain their bitcoins through trading with users that already own some BTC. These transactions mostly take place on bitcoin exchanges that provide a platform for the buying and selling of BTC for fiat currency. By looking at the prices at which BTC is traded on these exchanges, the "real-world value" of bitcoin can be deduced.

2 Bitcoin Reference Rate

The CME CF Bitcoin Reference Rate (BRR) is intended to facilitate financial products based on bitcoin. For instance, it may serve as a reference rate in the settlement of financial derivatives based on the bitcoin price, or in the net asset value (NAV) calculation of exchange traded funds (ETFs). It therefore must satisfy the Relevance, Manipulation Resistance, Verifiability and Replicability criteria whilst Timeliness is less important. Where possible, the BRR should also exhibit Stability and Parsimony.

2.1 Methodology

The BRR is to be calculated from trades data from a 1 hour period observation period. It is chosen to be an un-weighted mean of the average BTC:USD prices for each of 12 partition prices within the period. Partition prices are defined as the size-weighted median price for all trades executed during the partition. Formally, this is represented in Eq. 1.

Symbol	Name	Description
T	Effective Time	Time at which the BRR is calculated
τ	TWAP Period Length	Length of the Time Weighted Average Price (TWAP) period for which trade data is observed (60 minutes)
$\hat{\tau}$	Partition Length	Length of each partition (5 minutes)
x_i	Partition Trade	i^{th} price/size trade pair in partition
p_i	Partition Trade Price	i^{th} trade price in partition
s_i	Partition Trade Size	i^{th} trade size in partition
K	Number of Partitions	number of partitions – given by $\tau/\hat{\tau}$
k	Partition Number	k^{th} partition
WM_k	Weighted Median	Size-weighted median for the k^{th} partition
BRR_T	Bitcoin Reference Rate	BRR at time T

Figure 1: Meaning of symbols used in BRR methodology

$$
\begin{gathered}
WM_k = p_j \text{ where } x_j \text{ satisfies } \sum_{i=0}^{j-1} s_i < \frac{\sum_{i=1}^{I_k} s_i}{2} \text{ and } \sum_{i=j+1}^{I_k} s_i \leq \frac{\sum_{i=1}^{I_k} s_i}{2} \\
\text{If } \sum_{i=j+1}^{I_k} s_i = \frac{\sum_{i=1}^{I_k} s_i}{2} \text{ then } WM_k = \frac{p_j + p_{j+1}}{2} \\
BRR_T = \frac{\sum_{k=1}^{K} WM_k}{K}
\end{gathered}
\tag{1}
$$

WM_k is thus calculated as the price (p_j) of the j^{th} trade where the j^{th} trade is the trade that lies at 50% of the cumulative size for the partition k. WM_k is calculated for each partition in T and the BRR is found to be the mean WM_k of all the K partitions.

2.2 Analysis of Characteristics

2.2.1 Relevance

The BRR satisfies the criteria for relevance by definition as it represents actual trades that occurred. If the "true value" of the underlying asset is defined as the price it can be bought and sold at, then using completed trade data allows the BRR to provide an accurate reference to the average price over the period. The BRR does not necessarily represent the price at which the asset can be traded at the point of publication.

2.2.2 Manipulation Resistance

The chosen specification makes the BRR highly resistant against manipulation. The use of medians reduces the effect of outlier prices on one or more exchanges. Volume-weighting of medians filters out high numbers of small trades that may otherwise dominate a non volume-weighted median. The use of 12 non-weighted partitions assures that price information is sourced equally over the entire observation period. Influencing the BRR would therefore require price manipulation on several exchanges over an extended period of time.

2.2.3 Verifiability

The data for this calculation is readily available and the rate may be straightforwardly calculated given the methodology. The process has a low technical overhead and is thus readily verifiable.

2.2.4 Replicability

The BRR represents a mean value of the median trade price (weighted by size) during each sub-period. Thus, the result calculated for each sub-period represents (or is at least close to) an actual trade price executed on a specific CE. Whilst this CE is, in theory, unknown before the sub-period, it is likely that specific CEs will contain the volume-weighted median more regularly than others given the current price levels at different exchanges. A trader can therefore replicate the BRR by trading on the exchange(s) that trade close to the median and see a good agreement with the index. Further, equally-weighting partitions assures that the trader is able to determine exactly the size that needs to be transacted in each partition.

2.2.5 Timeliness

Whilst the BRR is published at a specific point in time, it is calculated over a certain (past) time period. Each publication of the BRR thus represents an average of the trading activity during a past period rather than the instantaneous rate. Calculation and then publication of the value of the BRR can feasibly be done in a matter of seconds; however, as the rate is not intended to represent the immediate price, its publication may be delayed without compromising its usefulness.

2.2.6 Stability

The BRR does not exhibit price fluctuations away from the true value of the underlying asset by virtue of being calculated from real trade data, and making use of a median, which results in a high resistance to outliers. This is only true in the absence of data quality issues, which can affect the rate in both directions.

2.2.7 Parsimony

The BRR uses very few parameters, none of which can be deemed superfluous. The only parameters required to calculate the rate are the period length τ and the number of partitions K.

2.3 Graphical Analysis

The BRR was calculated for a number of "distressed periods" that contained significant movements in the trading price of BTC. Each graph contains a plot for the calculated BRR based on all data as well as lines for the volume weighted average price on each consituent exchange. These results are intended to show the effect of the BRR methodology including the resilience to fluctuations in individual exchanges.

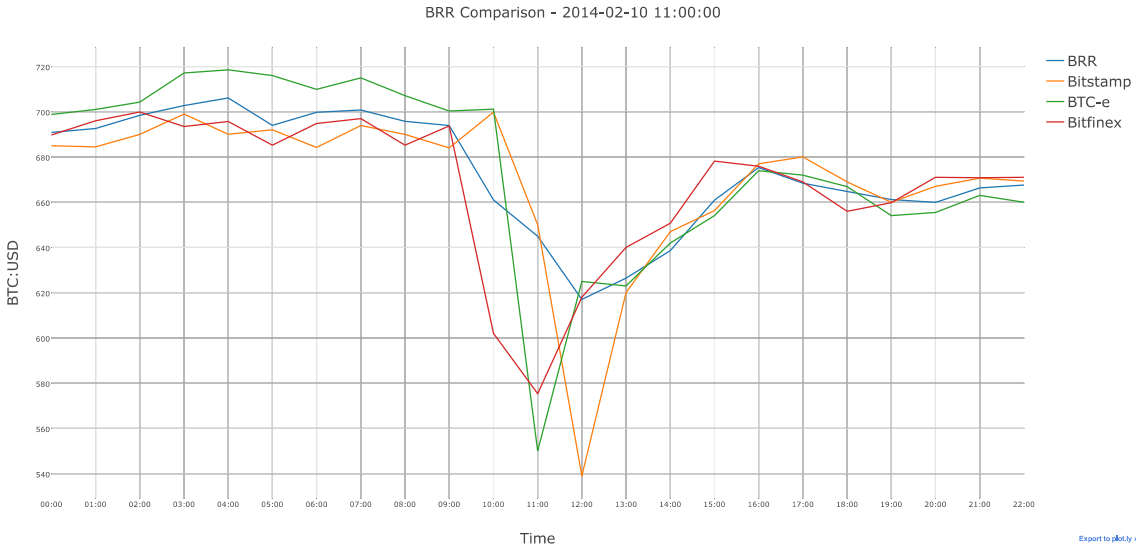

Figure 2: The performance of the BRR plotted against the VWAP (one-hour time horizon) of each constituent exchange for a sample observation period

As can be seen in Figure 2, the BRR is largely resistant to fluctuations of the prices on individual exchanges. Looking at 11:00, there was a significant drop in the price on Bitfinex followed by another at 12:00 on Bitstamp. The BRR can be seen to be resistant to both of these events, keeping more in line with the slower decline of the other CEs. Additional plots may be found in Appendix B.

3 Real Time Index

The CME CF Bitcoin Real Time Index (BRTI) is intended to serve as a live rate of the instantaneous US Dollar price of bitcoin. The BRTI must satisfy the Timeliness and Martingale Property criteria and should exhibit Stability, Parsimony and Manipulation Resistance. The index will be calculated and disseminated every second.

3.1 Specification

The BRTI will use the order books from a number of exchanges. These are combined every second to form the consolidated order book so that the index relies only on live data. The consolidated order book is sorted ascending in price for Asks and descending in price for Bids. The BRTI will be calculated as given in (2).

Symbol	Name	Description
T	Effective Time	Time at which the BRTI is calculated
v_{max}	Maximum Volume	Maximum volume of v used (set at 5000)
λ	Lambda	Parameter to determine shape of PDF (set as $1/0.3v_{\max}$)
v	Volume	Independent variable for price-volume curve
$ap_{T,i}$ and $bp_{T,i}$	Ask and Bid Price	Price for the i^{th} Ask and Bid respectively
$a\dot{s}_{T,i}$ and $b\dot{s}_{T,i}$	Normalised Ask and Bid Size	Size for the i^{th} Ask and Bid respectively, up to a maximum of 100
NF	Normalisation Factor	Parameter to normalise the PDF to 1 over the interval (0, $v_{\max}$)
$BRTI_T$	BRTI	The BRTI at effective time T

Figure 3: Meaning of symbols used in BRTI methodology

$$
\begin{aligned}
askPV_T(v) &= ap_{T,j+1} \text{ where } \sum_{i=1}^{j} a\dot{s}_{T,i} < v \text{ and } \sum_{i=1}^{j+1} a\dot{s}_{T,i} \geq v \\
bidPV_T(v) &= bp_{T,j+1} \text{ where } \sum_{i=1}^{j} b\dot{s}_{T,i} < v \text{ and } \sum_{i=1}^{j+1} b\dot{s}_{T,i} \geq v \\
midPV_T(v) &= \frac{askPV_T(v) + bidPV_T(v)}{2}
\end{aligned}
\tag{2}
$$

$$
BRTI_T = \int_0^{v_{\max}} midPV_T(v) \times \frac{1}{NF}\lambda e^{-\lambda v} dv \approx \sum_{v=1}^{v_{\max}} midPV_T(v) \times \frac{1}{NF}\lambda e^{-\lambda v}
$$

This methodology produces a pair of curves: $askPV_T(v)$ and $bidPV_T(v)$. The curve $midPV_T(v)$ is given by the average of these two curves for every v, including situations where $askPV_T(v)$ is lower than $bidPV_T(v)$. The product of the $midPV_T(v)$ curve and the PDF is integrated discretely over the maximum volume interval to compute $BRTI_T$.

3.2 Analysis of Characteristics

3.2.1 Timeliness

The BRTI does not rely on any historical data but is instead based on order book data. This makes it entirely forward looking in that it reflects prices at which one can trade now or in the future, as opposed to prices at which trades occurred in the past.

3.2.2 Martingale Property

The absence of a look-back period, for instance for the purpose of volume-weighting, assures that the BRTI is not susceptible to changes in response to data falling out of a look-back period. It therefore fulfills the martingale property, meaning that the best prediction of its future price is its current price.

3.2.3 Relevance

The use of order book data means that the BRTI represents interest to buy or sell rather than reflecting actual trades. The use of a mid price-volume curve in combination with the exponential PDF puts higher weight on bids and asks close to the global best bid and ask, which are most likely to be executed.

3.2.4 Verifiability and Replicability

The success of this report in calculating the BRTI for certain distressed periods (see below) illustrates its verifiability and replicability. However, due to the lack of historical data, this would not have been possible without access to historical order book data provided by Crypto Facilities. If the application of the methodology were to be challenged, the data would need to also have been independently collected to allow full recalculation of BRTI values.

3.2.5 Stability

By using a consolidated order book from several exchanges, the BRTI manages to capture a large amount of interest to buy and sell, increasing the stability of the index. However, as each CE differs slightly in its trading patterns, the loss of any CE can potentially cause fluctuations in the value of the index.

3.2.6 Parsimony

The formula for the BRTI is simple enough to recalculate and uses very few "magic numbers." Parameters have an intuitive basis in real world amounts and mainly serve to bound and normalise the calculation.

3.2.7 Manipulation Resistance

The use of an exponentially decaying PDF means that the BRTI is necessarily expensive to change significantly. Due to the heavy weighting of the exponential PDF, only bids and asks placed close to the global best bid and best ask have a significant effect on the BRTI, and these run a high risk of being executed. However, in times of low trading activity it may be possible to place large orders close to the best bid and ask whilst running a lower risk of these orders being executed. The low potential for affecting the index through "spoof" trades is illustrated in Figure 5 – the BRTI was calculated when a large (500BTC) order was placed at various points in the consolidated order book. The difference in the BRTI after placing these orders is almost undetectable and for the modifications over $1 away from the best bid or ask, the BRTI and modified BRTI are completely indistinguishable.

3.3 Numerical Results

The BRTI was calculated for a number of "distressed periods" that contained significant movements in the trading price of BTC, selected as 10 minute periods. Each graph contains a plot for the calculated BRTI based on all data as well as lines for the best bid on the constituent exchanges. These results are intended to demonstrate the characteristics of the BRTI methodology. Additional plots may be found in Appendix C.

As shown in Figure 4, the BRTI is resistant to significant changes on a single exchange. At the start of the period, Bitfinex shows a significant decline compared to the other CEs whilst towards the end of the period GDAX can be seen to have an increase over the other CEs. The BRTI is shown to be resistant to both of these outlier values, remaining in line with the majority of other CEs. By virtue of being formed from a consolidated order book, the BRTI updates much more granularly than any single CE.

4 Conclusion

The CME CF Bitcoin Reference Rate and CME CF Bitcoin Real Time Index show promise as, respectively, historical and real-time reference indices for bitcoin. Both have clear, reproducible methodologies and both exhibit desirable characteristics compatible with their intended use cases. We have been able to independently verify reference results across a wide range of scenarios. As with any financial index, problems may potentially arise due to data quality issues, exchange feed availability and at times of low trading activity. However, it is believed that these will be less impactful as the marketplace matures.

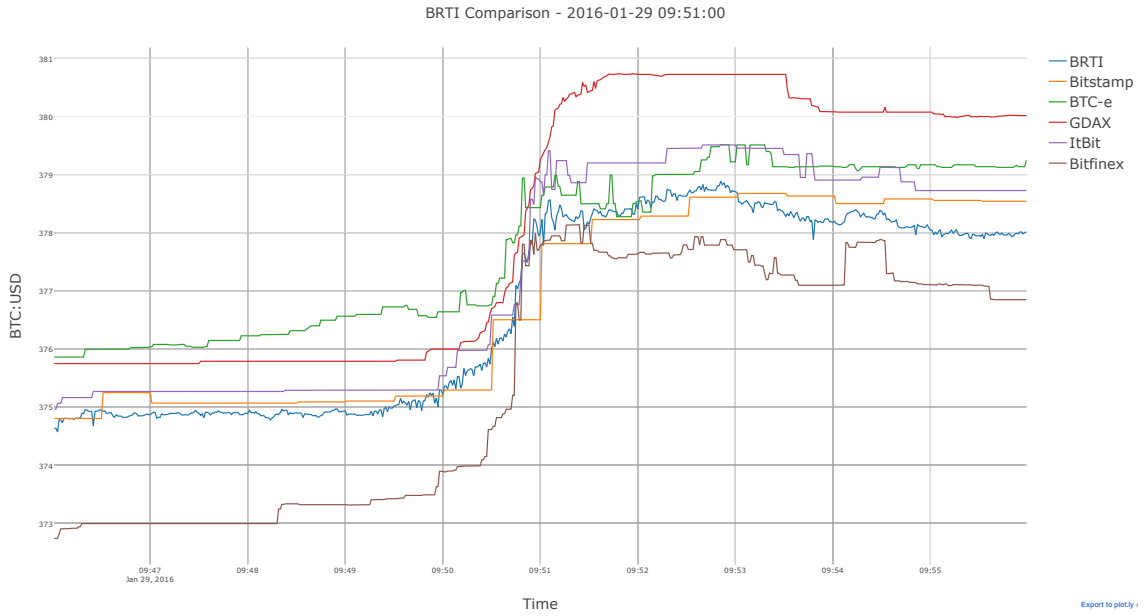

Figure 4: The performance of the BRTI plotted against the best mid for each constituent exchange

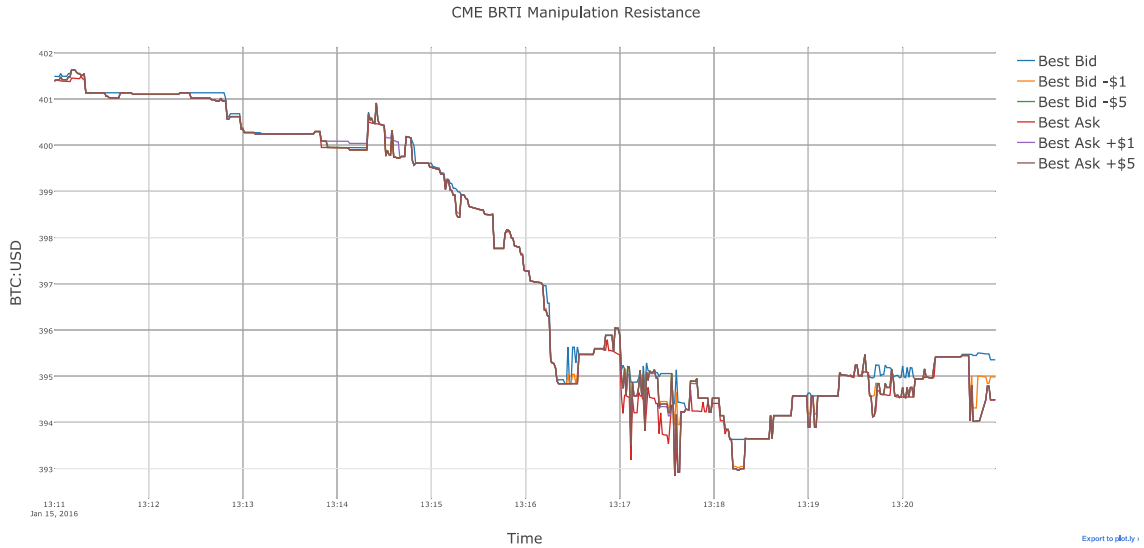

Figure 5: The BRTI with an order of 500BTC placed at different points in the consolidated order book

Time	Bitstamp	BTC-e	GDAX	ItBit	Bitfinex
15/01/2016 13:11	-0.358	0.447	-0.366	-0.108	0.0214
15/01/2016 13:43	-0.251	-0.405	-0.838	0.111	-0.051
15/01/2016 22:50	-0.319	0.494	-0.801	-0.541	0.035
20/01/2016 23:29	-0.317	0.057	0.000	-0.183	0.332
20/01/2016 23:39	-0.143	0.267	0.067	-0.169	0.201
24/01/2016 09:33	-0.305	0.244	-0.029	-0.130	0.055
29/01/2016 09:51	0.025	0.268	0.381	0.150	-0.330
21/02/2016 12:14	0.029	-0.167	-0.191	-0.350	0.483
21/02/2016 12:37	-0.095	-0.255	-0.395	-0.490	0.145

Table 1: Average percentage difference between the BRTI and the best mid for each CE

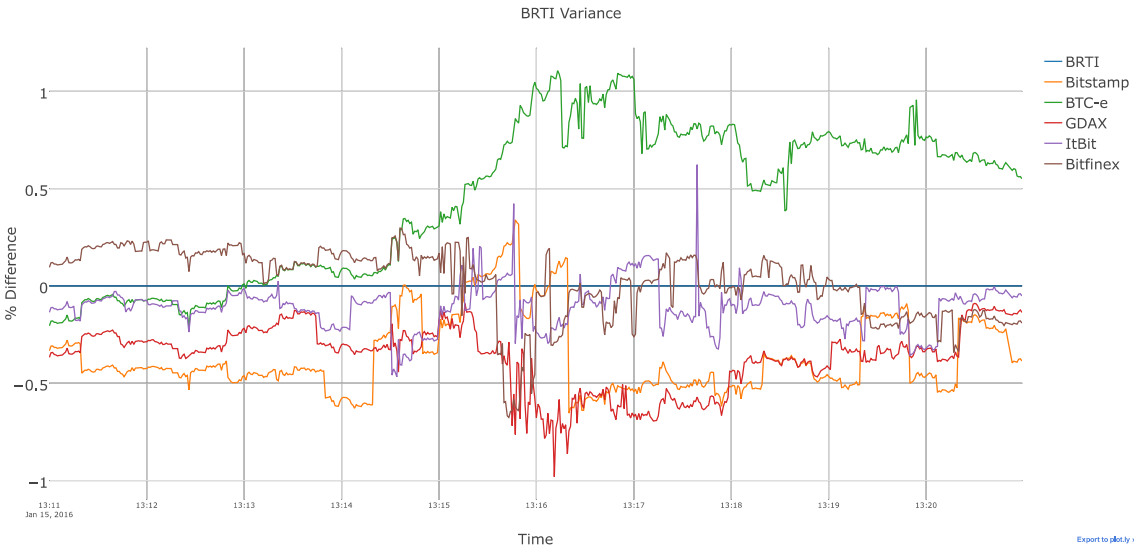

Figure 6: The percentage difference between the BRTI and the constituent exchanges

A BRR Additional Graphs

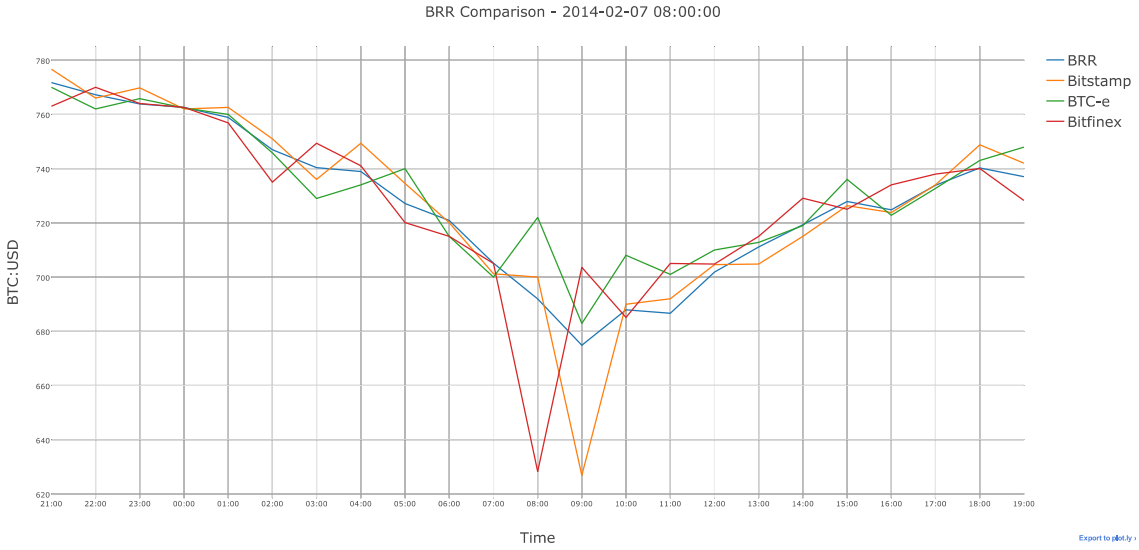

Figure 7: BRR resistant to strong temporary declines on on Bitfinex and Bitstamp

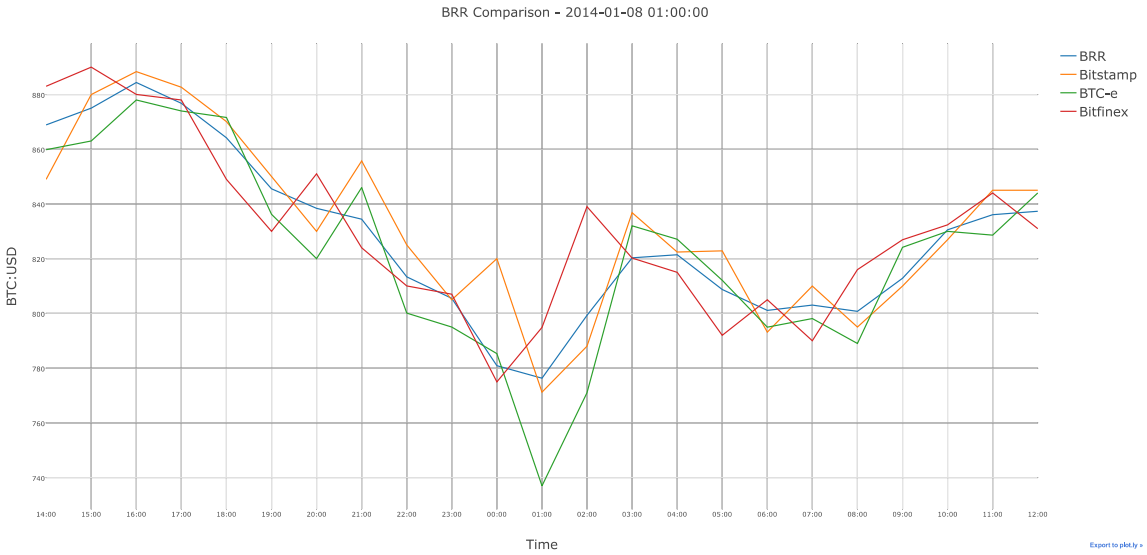

Figure 8: BRR resistant to strong temporary decline on BTC-e and consistent with other CEs

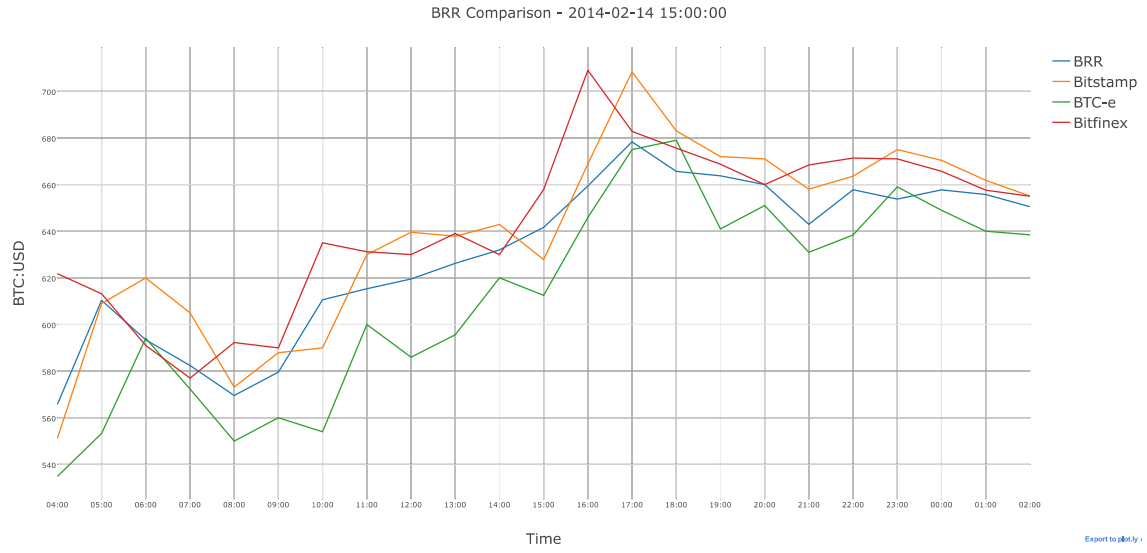

Figure 9: BRR resistant to outliers on BTC-e and Bitfinex

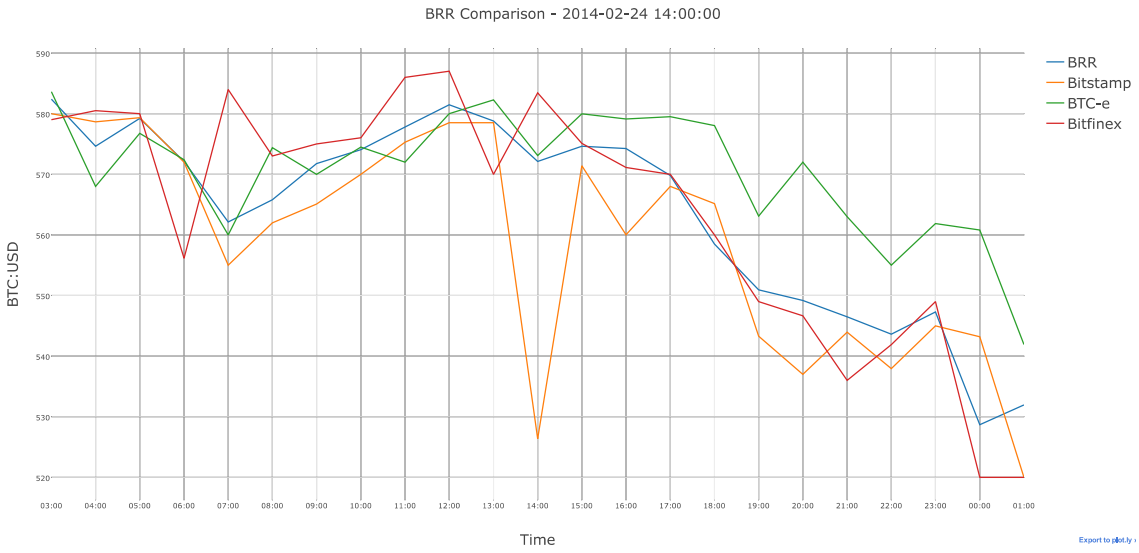

Figure 10: BRR resistant to strong temporary decline on Bitstamp and divergence of BTC-e

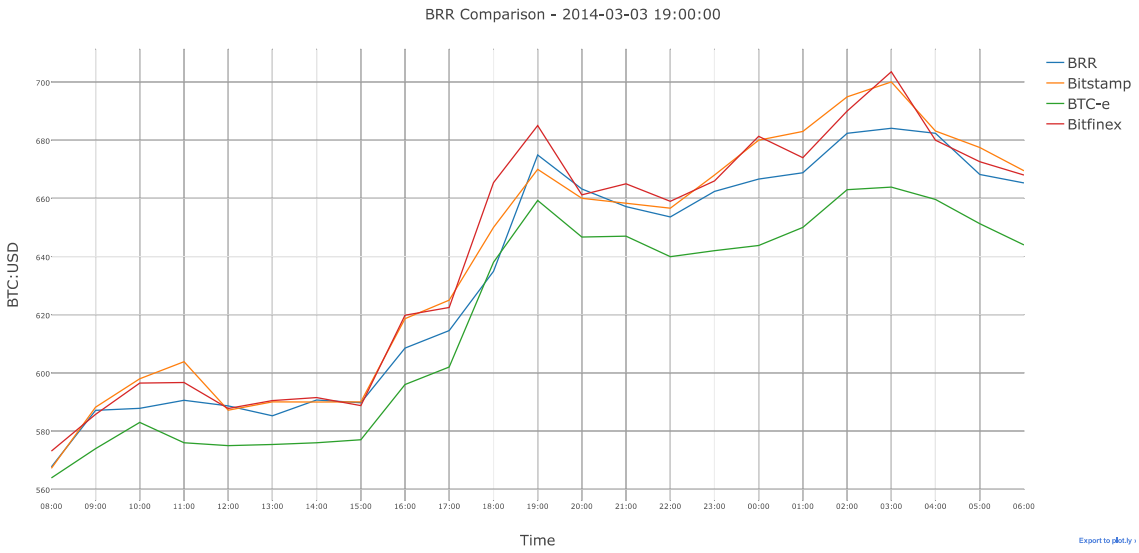

Figure 11: BRR resistant to divergence of BTC-e

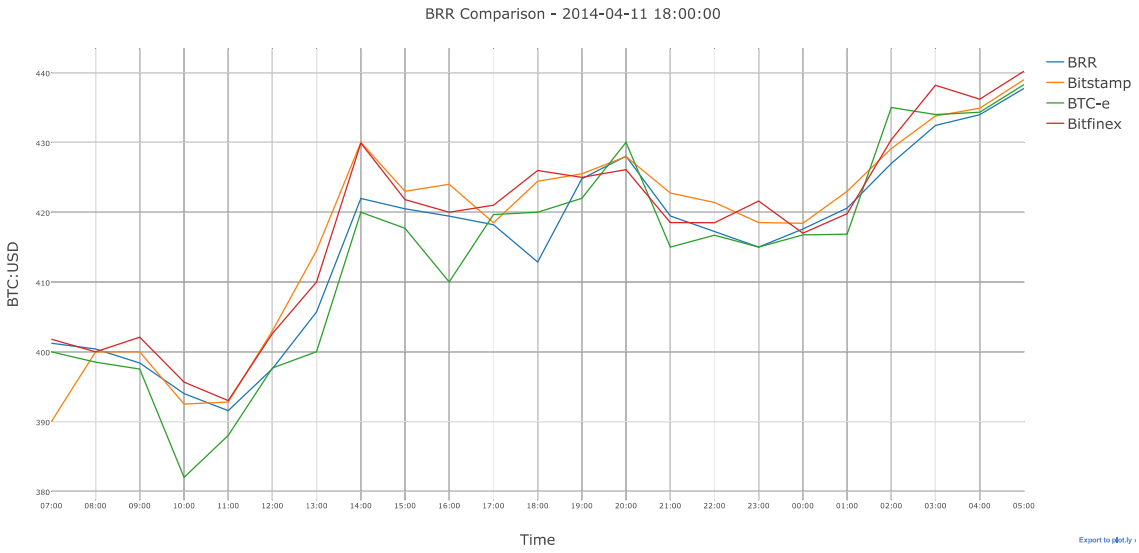

Figure 12: BRR consistent with all CEs

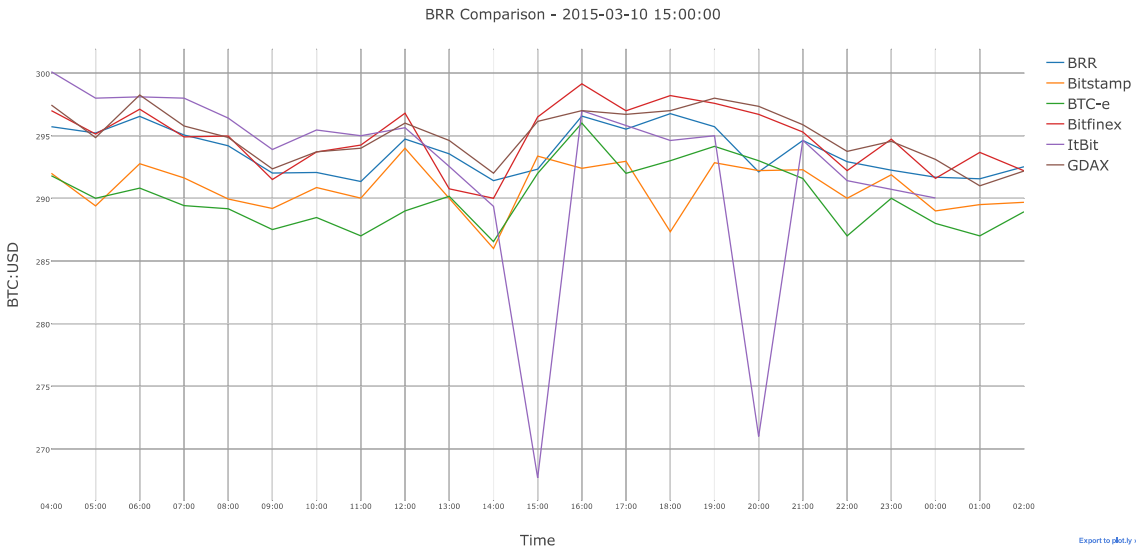

Figure 13: BRR resistant to multiple temporary declines on ItBit

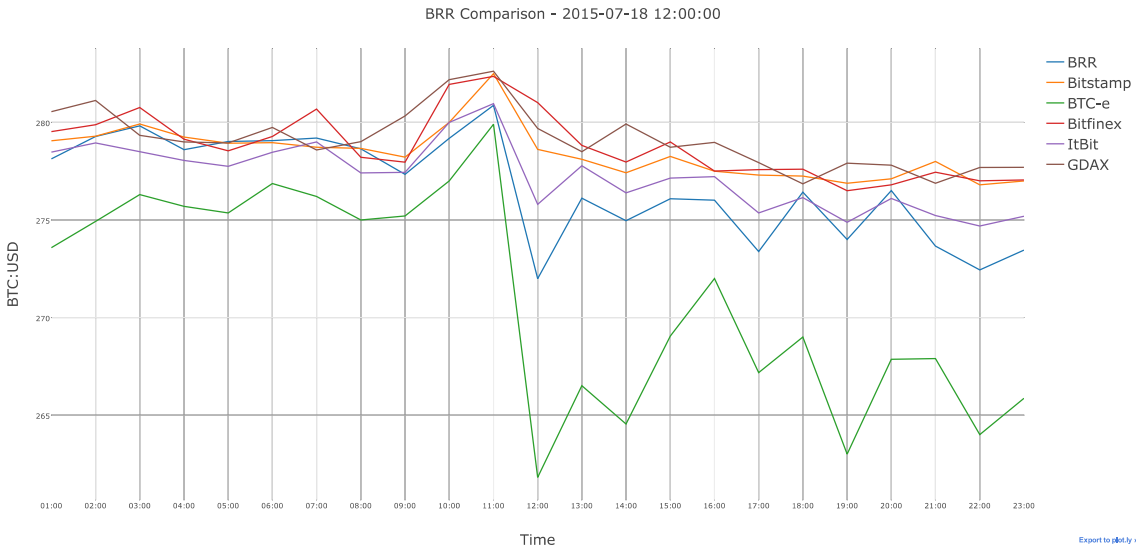

Figure 14: Large decline on BTC-e causes lesser decline on BRR

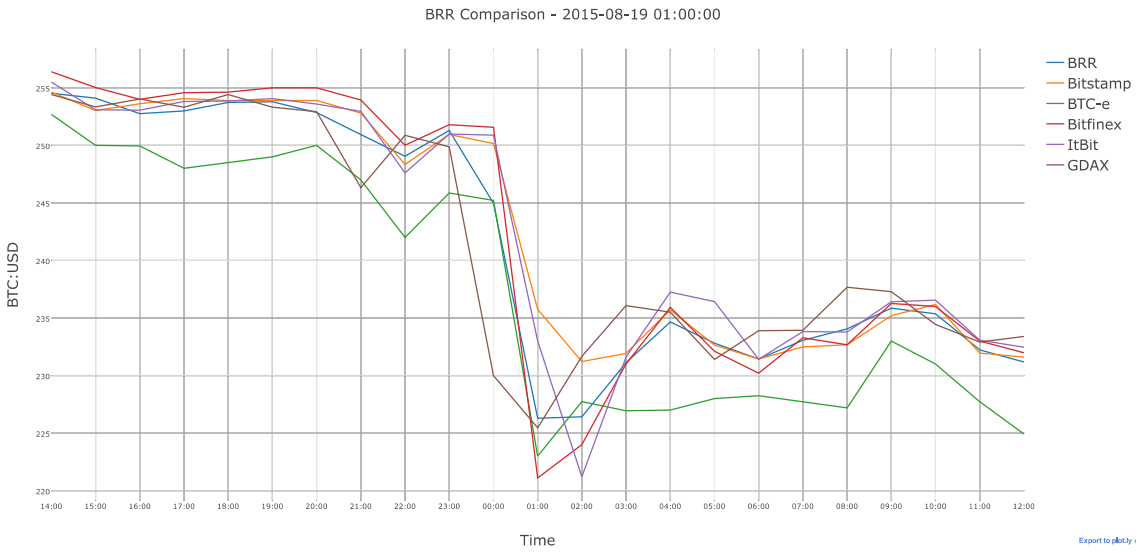

Figure 15: BRR consistent with all CEs

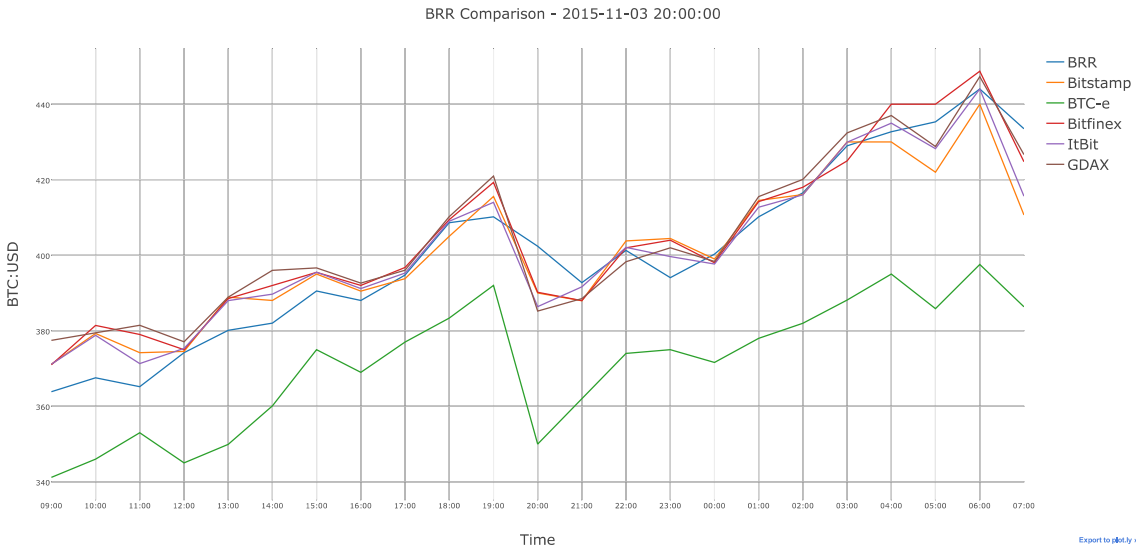

Figure 16: BRR resistant to divergence of BTC-e

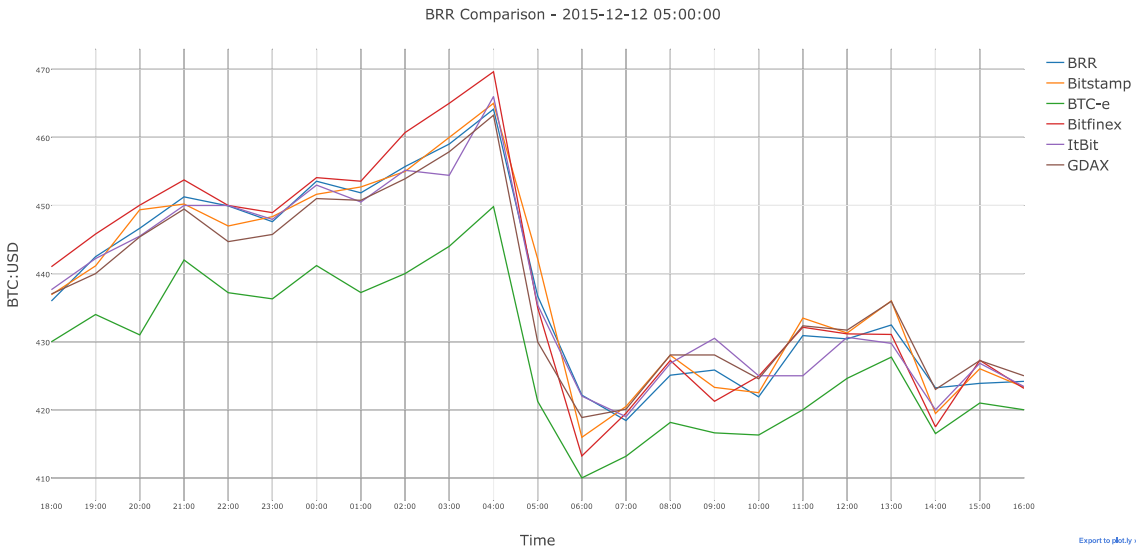

Figure 17: BRR resistant to divergence of BTC-e

B BRTI Additional Graphs

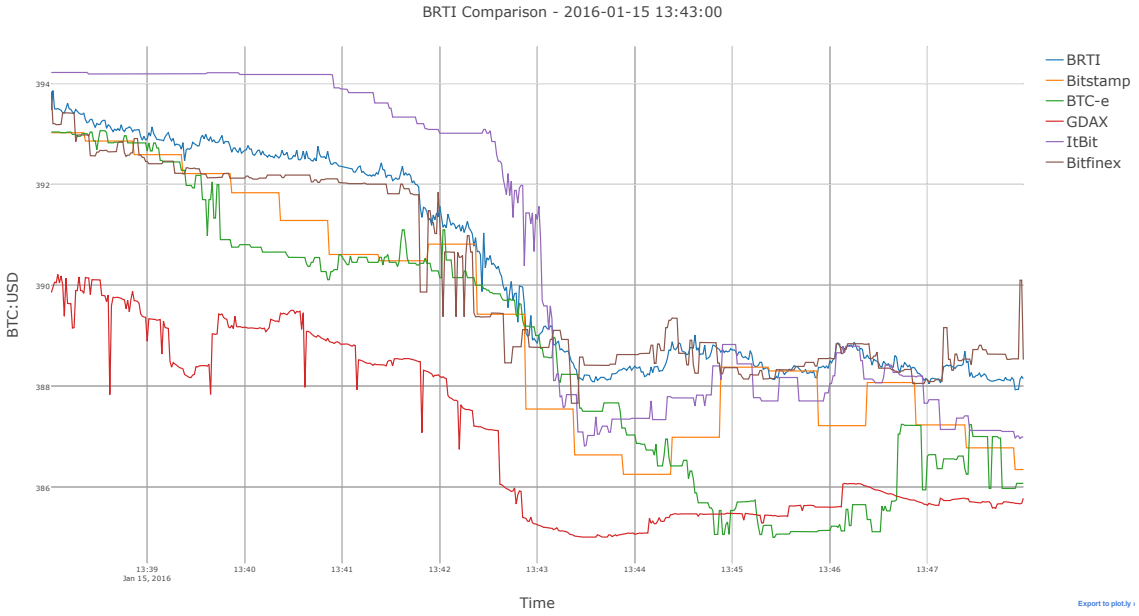

Figure 18: BRTI resistant to divergence of GDAX and BTC-e

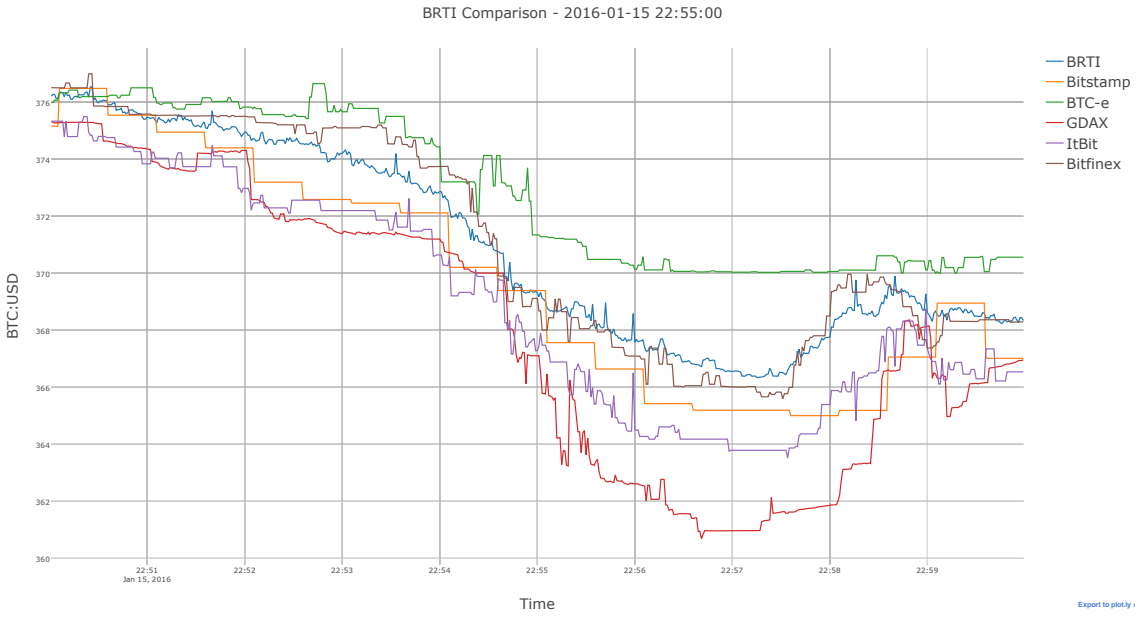

Figure 19: BRTI resistant to decline on GDAX

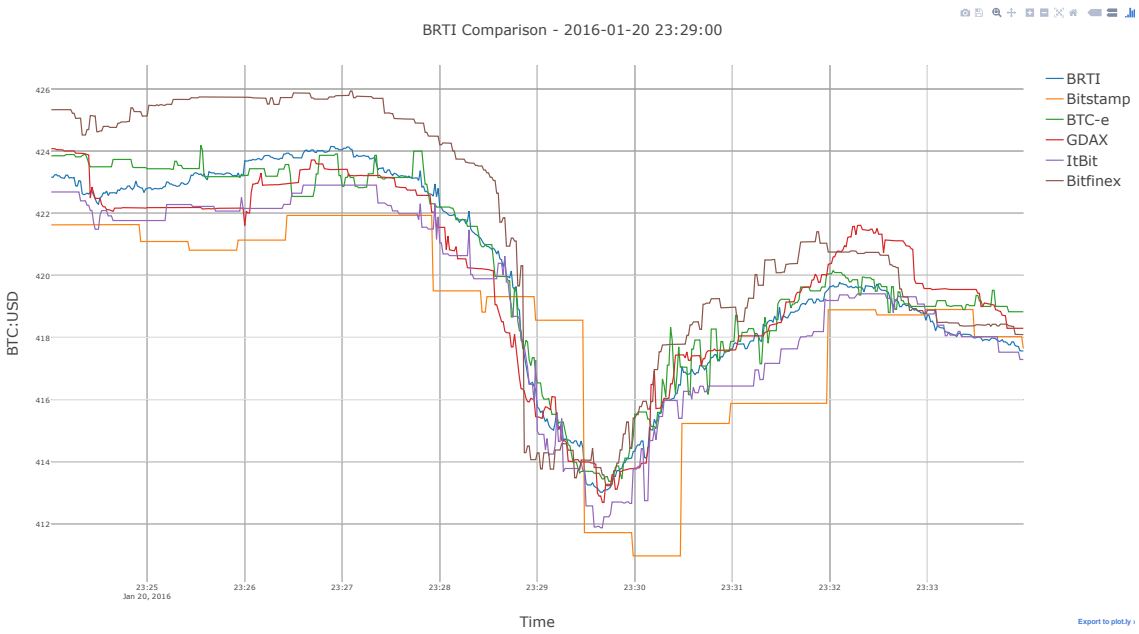

Figure 20: BRTI consistent with all CEs

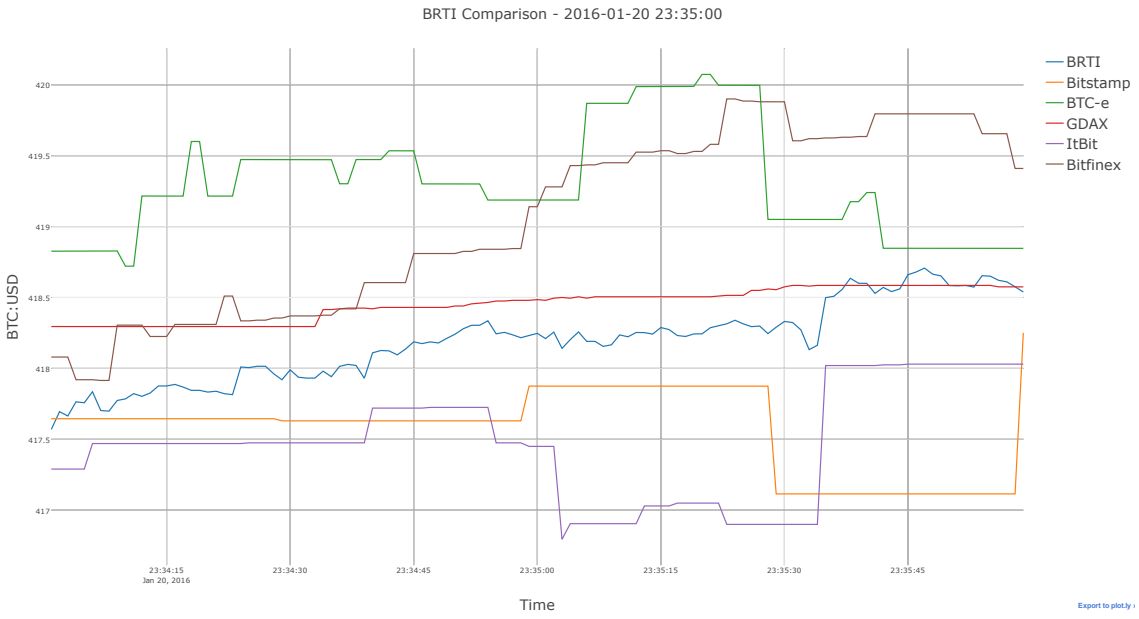

Figure 21: BRTI consistent with all CEs

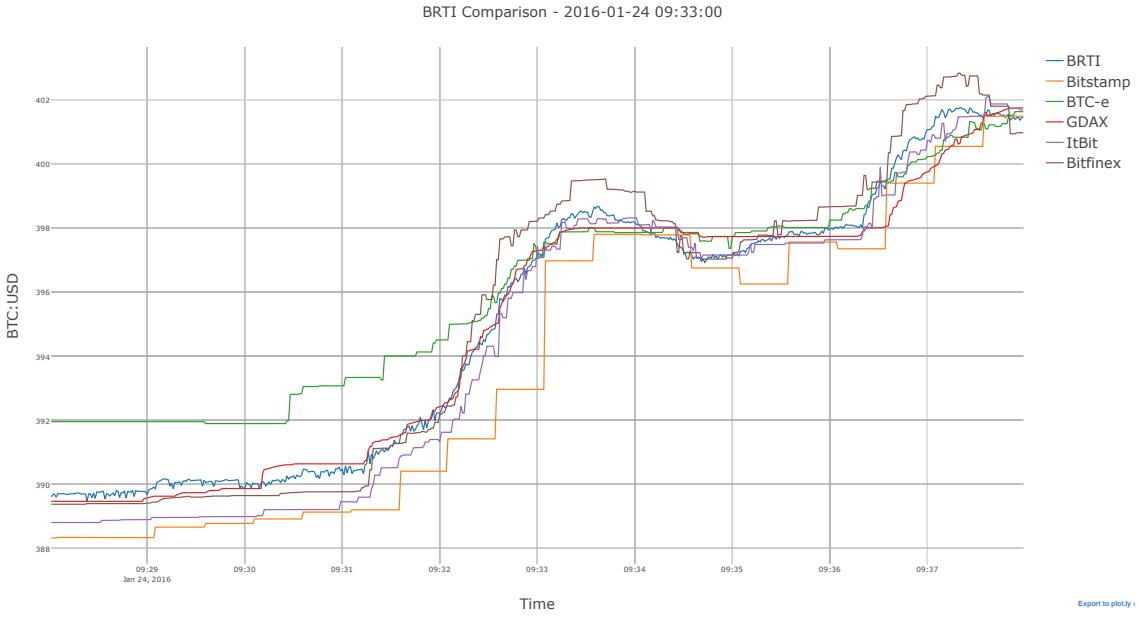

Figure 22: BRTI resistant to divergence of BTC-e

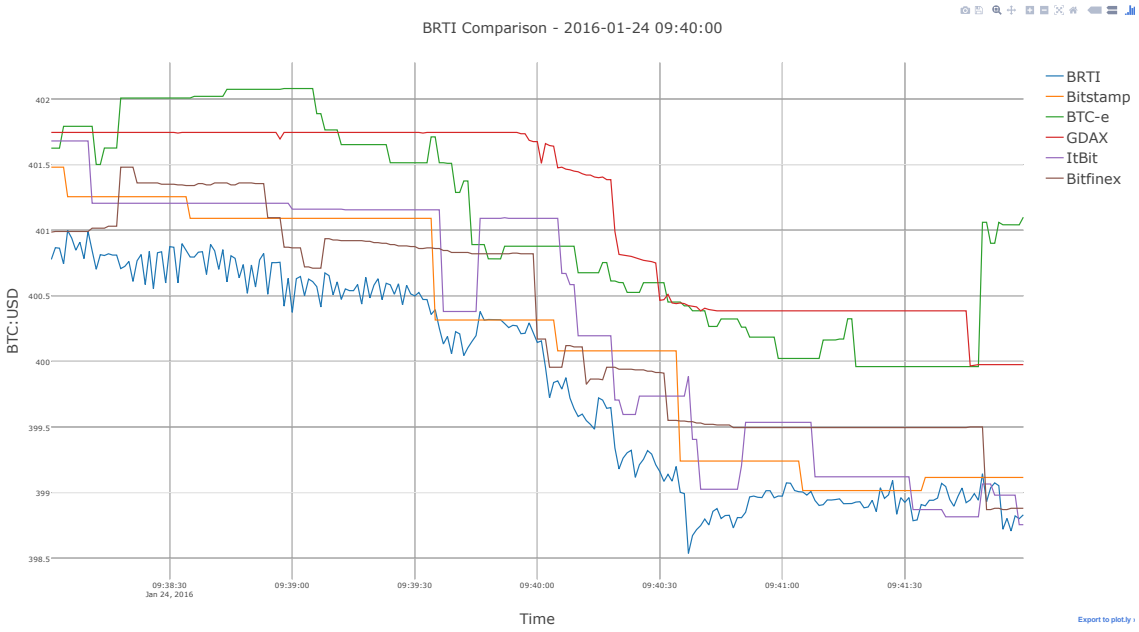

Figure 23: BRTI below all CEs – since results for CEs represent mid prices, this suggests that the ask volume in the consolidated orderbook substantially exceeds the bid volume

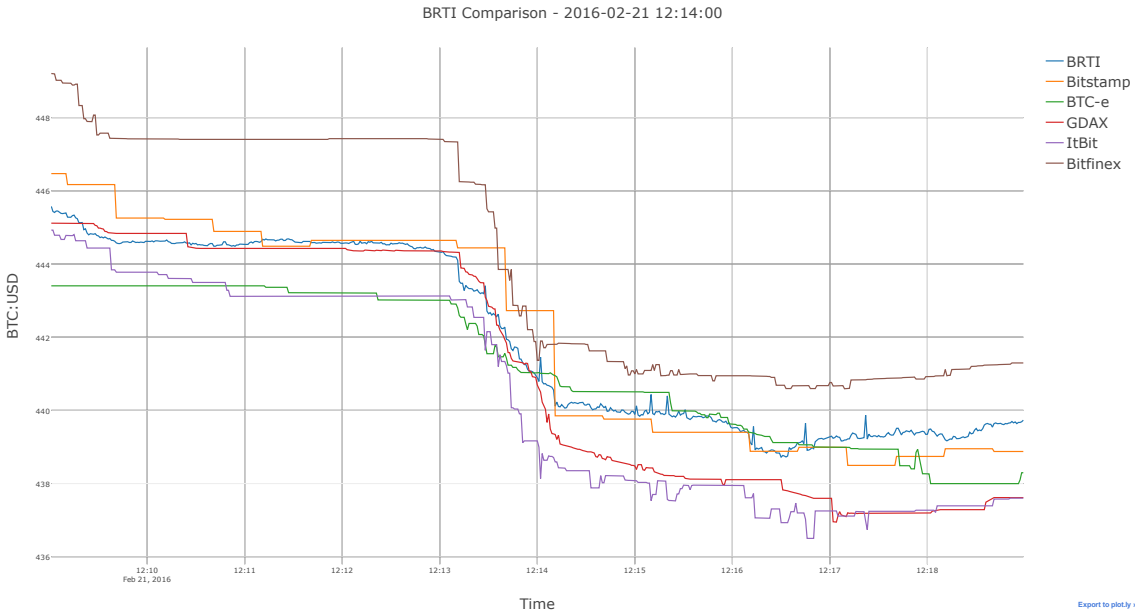

Figure 24: BRTI resistant to divergence of Bitfinex

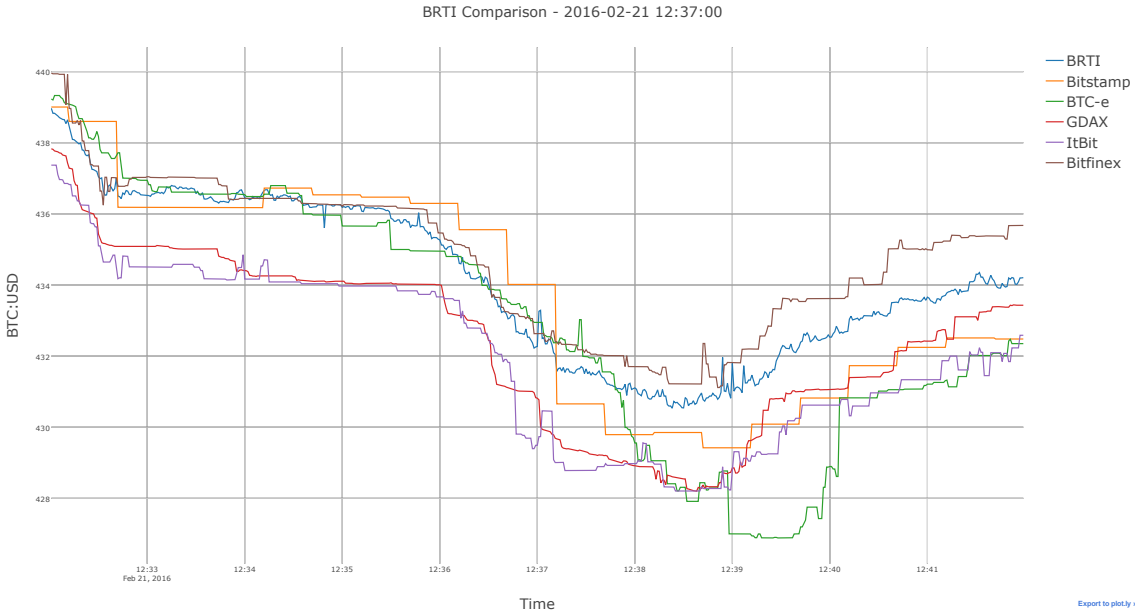

Figure 25: BRTI resistant to temporary divergence of BTC-e