Evolving Dynamic Trade Execution Strategies Using Grammatical Evolution

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Abstract. Although there is a plentiful literature on the use of evolutionary methodologies for the trading of financial assets, little attention has been paid to potential use of these methods for efficient trade execution. Trade execution is concerned with the actual mechanics of buying or selling the desired amount of a financial instrument of interest. Grammatical Evolution (GE) is an evolutionary automatic programming methodology which can be used to evolve rule sets. In this paper we use a GE algorithm to discover dynamic, efficient, trade execution strategies which adapt to changing market conditions. The strategies are tested in an artificial limit order market. GE was found to be able to evolve quality trade execution strategies which are highly competitive with two benchmark trade execution strategies.

1 Introduction

Grammatical Evolution is an Evolutionary Automatic Programming (EAP) technique which allows the generation of computer programs in an arbitrary language. GE can conduct an efficient exploration of a search space, and notably permits the incorporation of existing domain knowledge in order to generate 'solutions' with a desired structure. In finance (for example), this allows the users to seed the evolutionary process with their current trading strategies in order to see what improvements the evolutionary process can uncover. Recently GE has been successfully applied to a number of financial problems. These include financial time series modelling, intraday financial asset trading, corporate credit rating, and the uncovering of technical trading rules [2,16].

Trade execution is the process of trading a particular instrument of interest. A practical issue in trade execution is how to trade a large order as efficiently as possible. For example, trading of a large order in one lot may produce significant market impact costs. Conversely, by dividing an order into smaller lots and spreading these over time, a trader can reduce market impact cost but increases the risk of suffering opportunity cost. An efficient trade execution strategy seeks to balance out these costs in order to minimise the total trade cost. In this paper,

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GE is used to discover a dynamic trade execution strategy adapting to changing market conditions, while balancing off market impact cost and opportunity cost.

Instead of optimizing strategies by back-testing them over and over on the same historical data, we novelly test them in an artificial limit order market. One advantage of doing this is that the strategies can interact with the changing market. An agent-based modelling approach is adopted to simulate the artificial limit order market.

This paper is organized as follows. The next section provides a brief synopsis of the typical operation of an electronic double auction marketplace; Section 3 discusses trade execution strategies using GE and describes our performance evaluation approach; Section 4 explains agent-based modeling and describes how we implement the artificial stock market used in this study; Section 5 provides our results, with conclusion and some future work being presented in the final section of this paper.

2 Background

Today most market places operate an electronic double auction *limit order book*. Traders can either submit *a limit order* or *a market order*. A market order is an order to buy or to sell a specified number of shares. It guarantees immediate execution but provides no control on its execution price. In contrast, a limit order is an order to buy or to sell a specified number of shares at a specified price. It provides control over its execution price but does not guarantee its execution.

Table 1. Order Book 1

Bid		**Ask**	
Shares	Prices	Prices	Shares
300	50.19	50.22	200
200	50.18	50.23	300
400	50.17	50.24	100
500	50.16	50.25	300
300	50.15	50.26	200
100	50.14	50.27	400

Table 2. Order Book 2

Bid		**Ask**	
Shares	Prices	Prices	Shares
300	50.19	50.22	**200**
500	50.18	50.23	300
400	50.17	50.24	100
500	50.16	50.25	300
300	50.15	50.26	200
100	50.14	50.27	400

Table 3. Order Book 3

Bid		**Ask**	
Shares	Prices	Prices	Shares
300	50.19	50.22	**100**
500	50.18	50.23	300
400	50.17	50.24	100
500	50.16	50.25	300
300	50.15	50.26	200
100	50.14	50.27	400

Table 1 shows a sample order book, where all the buy and sell orders are visible to traders in the market. It consists of two queues which store buy and sell limit orders, respectively. Buy limit orders are called *bids*, and sell limit orders are called *offers* or *asks*. The highest bid price on the order book is called *best bid*, and the lowest ask price on the order book is called *best ask*. The difference between best bid and best ask is called *bid-ask spread*. Prices on the order book are not continuous, but rather change in discrete quanta called *ticks*.

Limit orders on the order book are typically (depending on market rules) executed strictly according to (1) price priority and (2) time priority. Bid (ask) orders with higher (lower) prices get executed first with time of placement being used to break ties. A buy (sell) market order is executed at the best ask (bid) price. The limit order book is highly dynamic, because new limit orders will be added into the order book, and current limit orders will get executed or cancelled from the order book throughout the trading day. Table 2 shows the order book after a trader submits a buy limit order with 300 shares placed at price 50.18. Table 3 shows the order book after a trader submits a buy market order with 100 shares.

3 Evolving Dynamic Trade Execution Strategies

A trade execution strategy is a set of rules determining a number of trade execution components designed to minimize transaction cost. These components include number of orders to be submitted, size of each order, what type each order should be and when each order should be submitted to the market.

The total trading volume of the order to be traded is often expressed as a percentage of the *average daily volume* (ADV) of the stock [11]. An order of less than 5% of ADV can generally be traded over a day without using complex strategies. On the contrary, if the target volume is larger than 15% of ADV, it may require execution over several days in order to minimize market impact. Normally, 5-15% of ADV is a reasonable order size which could expect to be tradable over a day using appropriate trade execution tactics. In this paper, the trading horizon of all strategies is one trading day and hence we assume that the order size is of this magnitude.

We assume that the order to be traded consists of V shares. The order is sliced into N smaller child orders (each of which will be submitted to the market according to our trading strategy), with order size s_1, s_2, ..., s_N, where

$$V = \sum_{i=1}^{N} s_i$$

A time window of half an hour is adopted in evolving our trading strategies. We benchmarked the results from our evolved trading strategies against two simple execution strategies. One simple trade execution strategy is a pure market order strategy in which each child order is submitted as a market order every half hour. This strategy takes market liquidity immediately by crossing the bid-ask spread. The other benchmark trade execution strategy is a pure limit order strategy. Traders submit each child order as a limit order placed at the best price, and amend its price to best price at a fixed frequency until this order is fully executed or until the trading period expires. At the end of trading day, any unexecuted orders are traded by crossing the bid-ask spread in order to ensure order completion. For instance, a buy order s_n may be submitted to the market as a limit order placed at the best bid price with an amendment frequency of Δt minutes. If Δt minutes after submission, this limit order is not fully executed, it will be amended to the best bid price. This amendment process continues in Δt intervals up to the end of trading day, at which time the uncompleted order(s) are traded as market orders by crossing the bid-ask spread.

In the simple market order strategy, *order aggression* (crossing the bid-ask spread) happens immediately after order submission which guarantees order execution, at the cost of market impact. In the simple limit order strategy, order aggression happens at the end of trading period aiming to reduce market impact, at the risk of opportunity cost. A more sophisticated limit order strategy would allow for order aggression between these two extreme cases. A general limit order strategy is to cross the uncompleted limit order over the spread after submission but before the end of trading day. In our GE evolved strategies, the timing of order aggression is determined by an execution rule evolved using GE. At each amendment time (an integral multiple of Δt minutes after submission), if the market condition satisfies the condition of the execution rule, order aggression happens, otherwise, the uncompleted order is amended to the best price. In this paper, an amendment frequency of 10 minutes is adopted in all limit order strategies. The market variables representing the market condition are examined in the next section.

3.1 Information Indicators

There are a large number of studies in the literature analyzing the relationship between order placement and the information content of limit order books.

Table 4. Definitions of Market Variables

Variables	**Definitions**
BidDepth	Number of shares at the best bid
AskDepth	Number of shares at the best ask
RelativeDepth	Total number of shares at the best five ask prices divided by total number of shares at the best five bid and ask prices
Spread	Difference between the best bid price and best ask price
Volatility	Standard deviation of the most recent 20 mid-quotes
PriceChange	Number of positive price changes within the past ten minutes divided by the total number of quotes submitted within the past ten minutes

Traders are more willing to place market orders when the market depth on the same side of the order book is large. If the market depth on the opposite side is larger, traders prefer to submit limit orders [3,6,18,23]. The incoming limit orders will have lower execution probability, suffering higher non-execution risk. When the bid-ask spread widens, traders prefer to submit limit orders in order to avoid large bid-ask spread cost [3,6,17,18,22,23]. Prior research is inconclusive on the effect of market volatility. Pascual and Verdas [17] show that higher historic volatility suggests limit order submission in mid cap stocks, but the opposite phenomenon is observed in large cap stocks. Hall and Hautsch [10] observe an increase of all kinds of order submission during periods of high volatility. Ranaldo [18] supports an inverse relation between order aggression and volatility, while Lo and Sapp [14] report a positive relationship between order aggression and volatility. Cao et al. [3] find that volatility has a minimal effect on order aggression. Verhoeven [22] argues that greater price volatility implies that a trader has a greater chance of executing his order at a better price. Hence, prior literature suggests a range of possible explanatory variables, but indicates that we have an incomplete theoretical understanding of how these factors interact. This suggests that there will be particular utility for the application of evolutionary methods to uncover a suitable model structure (trade execution strategy). Based on the explanatory factors considered in the literature, we selected six information indicators to construct a dynamic trade execution strategy (Table 4).

3.2 Grammar of Grammatical Evolution Algorithm

The grammar adopted in our experiments is defined as follows:

```
<lc> ::= if (<stamt>)
                class = "CrossingSpread"
         else
                class = "NotCrossingSpread"
<stamt> ::= <cond1><op><cond2><op><cond3><op><cond4>
            <op><cond5><op><cond6>
<op> ::= and | or
<cond1> ::= (BidDepth>AvgBidDepth) is <boolean>
<cond2> ::= (AskDepth>AvgAskDepth) is <boolean>
<cond3> ::= (RelativeDepth>AvgRelativeDepth) is <boolean>
<cond4> ::= (Spread>AvgSpread) is <boolean>
<cond5> ::= (Volatility>AvgVolatility) is <boolean>
<cond6> ::= (PriceChange>AvgPriceChange) is <boolean>
<boolean> ::= True | False
```

In the grammar, $AvgBidDepth$ represents the average bid depth of the market, $AvgAskDepth$ represents the average ask depth of the market, $AvgRelativeDepth$ represents the average relative depth of the market, $AvgSpread$ represents the average spread of the market, $AvgVolatility$ represents the average volatility of the market and $AvgPriceChange$ represents the average price change of the market. The six financial variables are observed at the time of order amendment. An example of an evolved dynamic strategy using three financial variables is as follows.

```
if ( (BidDepth>AvgBidDepth) is True  or  (AskDepth>AvgAskDepth) is False
      and  (Spread>AvgSpread) is True  )   class = "CrossingSpread"
else        class = "NotCrossingSpread"
```

In this strategy, if the market condition satisfies

```
(BidDepth>AvgBidDepth) is True and (Spread>AvgSpread) is True
```

or satisfies

```
(AskDepth>AvgAskDepth) is False and (Spread>AvgSpread) is True
```

the uncompleted limit order will be crossed over the bid-ask spread. Otherwise, its limit price will be amended to the best price.

3.3 Performance Evaluation

The standard industry metric for measuring trade execution performance is the *VWAP measure*, short for *Volume Weighted Average Price*. It is calculated as the ratio of the value traded and the volume traded within a specified time horizon

$$VWAP = \frac{\sum(Volume * Price)}{\sum(Volume)}$$

where $Volume$ represents each traded volume and $Price$ represents its corresponding traded price. An example is shown in Figure 1.

	Submission Time	Shares		Traded Price		Value
Child Order 1:	t_0	400	$*$	50.15	=	20,060
		600	$*$	50.16	=	30,096
Child Order 2:	$t_1(t_0 + \Delta t)$	1,000	$*$	50.40	=	50,400
Child Order 3:	$t_2(t_0 + 2\Delta t)$	200	$*$	50.34	=	10,068
		800	$*$	50.36	=	40,288
Child Order 4:	$t_3(t_0 + 3\Delta t)$	1,000	$*$	50.39	=	50,390
Child Order 5:	$t_4(t_0 + 4\Delta t)$	1,000	$*$	50.68	=	50,680
Child Order 6:	$t_5(t_0 + 5\Delta t)$	1,000	$*$	51.10	=	51,100
Child Order 7:	$t_6(t_0 + 6\Delta t)$	1,000	$*$	50.87	=	50,870
Child Order 8:	$t_7(t_0 + 7\Delta t)$	700	$*$	50.98	=	35,686
		300	$*$	51.00	=	15,300
Child Order 9:	$t_8(t_0 + 8\Delta t)$	1,000	$*$	50.39	=	50,390
Child Order 10:	$t_9(t_0 + 9\Delta t)$	1,000	$*$	50.26	=	50,260
Total:		10,000				505,588
VWAP $= 505,588/10,000 = 50.5588$						

Fig. 1. VWAP Calculation of A Sample Buy Strategy

In order to evaluate the performance of a trade execution strategy, its VWAP is compared against the VWAP of the overall market. The rationale here is that performance of a trade execution strategy is considered good if the VWAP of the strategy is more favorable than the VWAP of the market within the trading period and bad if the VWAP of the strategy is less favorable than the VWAP of the market within the trading period. For example, if the VWAP of a buy strategy ($VWAP_{strategy}$) is lower than the market VWAP ($VWAP_{market}$), it is considered as a good trade execution strategy. Conversely, if the $VWAP_{strategy}$ is higher than the $VWAP_{market}$, it is considered as a bad trade execution strategy. Although this is a simple metric, it largely filters out the effects of volatility, which composes market impact and price momentum during the trading period [1]. The performance evaluation functions for each trading day are as follows:

$$VWAP\ Ratio = \begin{cases} \frac{10,000*(VWAP_{strategy} - VWAP_{market})}{VWAP_{market}} & \text{Buy Strategy} \\ \frac{10,000*(VWAP_{market} - VWAP_{strategy})}{VWAP_{market}} & \text{Sell Strategy} \end{cases}$$

where $VWAP_{market}$ is the average execution price which takes into account all the trades over the day excluding the strategy's trades. This corrects for bias, especially if the order is a large fraction of the daily volume [13]. For both buy and sell strategies, the smaller the VWAP Ratio, the better the strategy is.

4 Simulating an Artificial Market

In our experiments, the training and evaluation of all trade execution strategies are implemented in an artificial limit order market, which is simulated using an agent-based model.

Agent-based modelling is a computerized simulation consisting of a number of agents. The emergent properties of an agent-based model are the results of "bottom-up" processes, where the decisions of individual and interacting agent at a microscopic level determines the macroscopic behavior of the system. For a more detailed description of agent-based modelling in finance, please refer to [12,19,20]. In this paper, our agent-based artificial limit order market is built based on the *Zero-Intelligence* (ZI) model [5] with a continuous double auction price formation mechanism. The notion of ZI agents was first mentioned in Gode and Sunder [9]. These agents randomly generate buy and sell orders. The orders are then submitted to a market agent, who manages all incoming orders according to the order matching mechanism in a real limit order market. The trading process is continuous, where unmatched orders are stored in an order book.

At each time step, an agent is equally likely to generate a buy order or a sell order. This order can be a market order, or a limit order, or a cancellation of a previous order, with probabilities λ_m, λ_l, and λ_c respectively. The sum of these probabilities is one ($\lambda_m + \lambda_l + \lambda_c = 1$). For a limit buy (sell) order, it has a probability of $\lambda_{inSpread}$ falling inside the bid-ask spread, a probability of λ_{atBest} falling at the best bid (ask) price, and a probability of λ_{inBook} falling off the best bid (ask) price in the book, ($\lambda_{inSpread} + \lambda_{atBest} + \lambda_{inBook} = 1$). The limit price inside the spread follows a uniform distribution. The limit price off the best bid (ask) price follows a power law distribution with the exponent of $(1 + \mu_1)$. The log order size of a market order follows a power law distribution with the exponent of $(1 + \mu_2)$, while the log order size of a limit order follows a power law distribution with the exponent of $(1 + \mu_3)$.

As each incoming buy (sell) market order arrives, the market agent will match it with the best ask (bid) limit order stored in the order book. If this market order is fully filled by the first limit order, the unfilled part will be matched to the next best ask (bid) limit order until it is fully filled. As each incoming limit order arrives, the market agent will store it in the order book according to price

Table 5. Initial Parameters for Artificial Limit Order Market

Explanation	Value
Initial Price	$price^0 = 50$
Tick Price	$\delta = 0.01$
Probability of Order Cancellation	$\lambda_c = 0.34$
Probability of Market Order	$\lambda_m = 0.16$
Probability of Limit Order	$\lambda_l = 0.50$
Probability of Limit Order in Spread	$\lambda_{inSpread} = 0.32$
Probability of Limit Order at Best Quote	$\lambda_{atBest} = 0.33$
Probability of Limit Order off the Best Quote	$\lambda_{inBook} = 0.35$
Limit Price Power Law Exponent	$1 + \mu_1 = 2.5$
Market Order Size Power Law Exponent	$1 + \mu_2 = 2.7$
Limit Order Size Power Law Exponent	$1 + \mu_3 = 2.1$

and time priority. As each incoming cancelation order arrives, the market agent will delete the relevant limit order in the order book.

In order to ensure that the order flows generated by the artificial market are economically plausible, all the parameters in our model are derived from empirical evidence [4,7,8,15,21]. The parameters used in our simulation are presented in Table 5.

5 Results

In this study we consider a large order of 10% of ADV of the artificial market, which is to be traded over one day (5 hours in the artificial market). This order is equally divided into ten child orders. In all trade execution strategies, any uncompleted orders are crossed over the spread at the end of trading day in order to ensure order completion.

Our experiments comprise of two periods (training and test periods). In the training period, GE is used to evolve dynamic trade execution strategies. Each individual is exposed to 20 continuous trading days in the artificial market and their fitness is calculated as their average VWAP ratio over the 20 trading days. The GE experiment is run for 20 generations, with variable-length, one-point crossover at a probability of 0.9, one point bit mutation at a probability of 0.01, roulette selection, steady-state replacement and a population size of 100. In the test period, the best evolved strategy in the training period is tested out of sample over 240 days in the artificial market. The performances of simple market order strategies (SM) and simple limit order strategies (SL) are also evaluated in order to benchmark the GE strategies.

Table 6. Results of best evolved GE strategies and two benchmark strategies

	SM		**SL**		**GE**			
	Mean	(S.D.)	Mean	(S.D.)	Mean	(S.D.)	H_1	H_2
Buy Order	69.64	(0.42%)	42.54	(1.45%)	-1.42	(0.49%)	0.00	0.01
Sell Order	68.73	(0.36%)	13.81	(1.59%)	-23.21	(0.48%)	0.00	0.01

The results (all out of sample) of buy strategies and sell strategies are provided in Table 6. The "Mean" is the average VWAP ratio of each strategy over the 240 days, and "S.D." represents the standard deviation of the average (daily) VWAP ratio. P-values for the null hypothesis $H_1 : mean_{SM} \leq mean_{GE}$ and $H_2 : mean_{SL} \leq mean_{GE}$ are also shown in the table, to indicate the degree of statistical significance of the performance improvement of GE strategies over the two simple strategies. The figures show that the null hypotheses are rejected at the ≤ 0.01 level.

Based on the results, GE evolved strategies notably outperform the two benchmark strategies, simple market order strategy (SM) and simple limit order strategy (SL). The negative VWAP ratios of -1.42 and -23.21 show that the GE evolved strategies achieve better execution price than the average execution price of the market. The small standard deviations of 0.49 and 0.48 suggest that the applicability of GE for evolving quality dynamic trade execution strategies. Comparing the performance of the strategies for buy and sell orders, we observe that the performances of sell strategies are better than those of buy strategies.

6 Conclusions and Future Work

Trade execution is concerned with the actual mechanics of trading an order. Traders wishing to trade large orders face tradeoffs in balancing market impact and opportunity costs. Trade execution strategies are designed to balance out these costs, thereby minimizing transaction cost relative to some benchmark like VWAP. Despite the importance of optimising trade execution, there has been relatively little attention paid in the literature to the application of evolutionary methods for this task. In this paper, GE was novelly applied for the purposes of evolving dynamic trade execution strategies, and an artificial limit order market was simulated for testing the evolved trade execution strategies. GE was found to be able to evolve quality trade execution strategies which proved highly competitive against two benchmark trade execution strategies.

There is notable scope for further research utilising GE in this problem domain. One obvious route is to widen the number of market variables which can be included in the evolved execution strategies. Another route is to evolve the full structure of the trade execution strategy. In our approach, we focused on one aspect of trade execution strategy (when to cross the spread), and other components like the number of orders are determined in advance. Future work will embrace the evolution of the full structure of trade execution strategy.

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