A statistical risk assessment of Bitcoin and its extreme tail behaviour

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Abstract

We provide an extreme value analysis of the returns of Bitcoin. A particular focus is on the tail risk characteristics and we will provide an in-depth univariate extreme value analysis. Those properties will be compared to the traditional exchange rates of the G10 currencies versus the US dollar. For investors - especially institutional ones - an understanding of the risk characteristics is of utmost importance. So for bitcoin to become a mainstream investable asset class, studying these properties is necessary. Our findings show that the bitcoin return distribution not only exhibits higher volatility than traditional G10 currencies, but also stronger non-normal characteristics and heavier tails. This has implications for risk management, financial engineering (such as bitcoin derivatives) - both from an investor's as well as from a regulator's point of view. To our knowledge, this is the first detailed study looking at the extreme value behaviour of the cryptocurrency Bitcoin.

1 Introduction

Invented by an unidentified programmer under the name of Satoshi Nakamoto, Bitcoin was introduced on October 31, 2008 to a cryptography mailing list, and released as open-source software in 2009 ((Nakamoto 2009)). Bitcoin is a form of cryptocurrency - a digital asset and an electronic payment system based on cryptographic proof, instead of traditional trust. The system is peer-to-peer and transactions take place between users directly, without an intermediary. These transactions are verified by network nodes and recorded in a public distributed ledger called the blockchain, which uses Bitcoin as its unit of account.

Bitcoin is the first decentralized digital currency and, as of November 2016, the largest of its kind in terms of total market value, representing over 81% of the total market value of cryptocurrencies (Source: www.coinmarketcap.com, retrieved November 1, 2016). As of October 2016, more than 720 cryptocurrencies exist. The second and third largest cryptocurrencies are Ethereum and Ripple, representing 7.6% and 2.4% of the market. The top ten of those 720 cryptocurrencies (Bitcoin, Ethereum, Ripple, Litecoin, Ethereum Classic, Monero, Dash, Augur, MaidSafeCoin, Waves) represent about 95% of the market.

Traditional financial markets are characterized by currency crises, stock market crashes, large credit defaults and other extreme events that might lead to large losses for investors. Cryptocurrencies show even larger volatility swings and extreme tail events. Using extreme value theory, we want to give a statistical characterization of the tail properties of Bitcoin returns. It is well-known that traditional fiat currencies and Bitcoin behave very differently from each other if volatility is used as metric, see e.g. (Sapuric and Kokkinaki 2014) and (Kristoufek 2015). The aim of this paper is to also investigate the tail behaviour of Bitcoin and contrast and compare it to the G10 currencies. There have been studies investigating the tail behaviour of currencies but none so far for the exchange rate of Bitcoin.

So far, research on Bitcoin and its risks has focused on security protocols, fraud and criminal activities, exchange defaults, money laundering, encryption techniques and similar topics. The risks of an investment in Bitcoin from a statistical point of view, the exchange rate fluctuations as well as the

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bevahiour of Bitcoin in extreme scenarios has not been investigated in great detail yet. We aim to close this gap by providing a detailed analysis of the statistical risks and extreme tail behaviour of the Bitcoin exchange rate. This paper is organized as follows. In section 2, we give an overview of the data which we used and the sources from which it was retrieved. Section 3, extreme value analysis is the main section and investigates the tails of the G10 currencies and bitcoin returns. Section 4 summarizes our findings.

2 Data

We are using historical global price indices for Bitcoin from the database Baverage from quandl.com, which shows aggregated Bitcoin price indices from multiple exchanges providing a weighted average Bitcoin price. Prices are regularly collected from different online exchanges, collated and a weighted average Bitcoin price for the different currencies is calculated based on activity, trading volume, liquidity and other factors. By means of this aggregation, the resulting time series better represents the price of Bitcoin in the global Bitcoin trading network. For the G10 currencies, we are using the database CurrFX, also from quandl.com. Daily data is downloaded from the beginning of September 2013 until the end of September 2016.

3 Extreme value analysis

Bitcoins are much more volatile than traditional fiat currencies. In the risk management of financial instruments, it is important to assess the probability of rare and extreme events. We will use extreme value theory to statistically model such events and compute extreme risk measures. Extreme value distributions will be fitted to the tails of Bitcoin returns and compared to the tails of traditional fiat currencies. We will also compute two tail risk measures, value at risk and expected shortfall for Bitcoin.

3.1 Motivation and normal distribution

Before we go into the extreme value behaviour of Bitcoin returns, we want to show the distribution of returns and compare them to a normal distribution. Deviations of the Bitcoin distribution from the normal distribution will justify our choice of using extreme value theory to describe the tails. In figure 1, we are showing the histogram of daily returns of the Bitcoin / USD exchange rate. The red line which is overlayed, shows a normal distribution with mean and standard deviation taken from the empirical Bitcoin/ USD exchange rates. The blue line is showing a Gaussian kernel-density estimator with bandwidth multiple 0.5. We see a substantial deviation from the normal distribution.

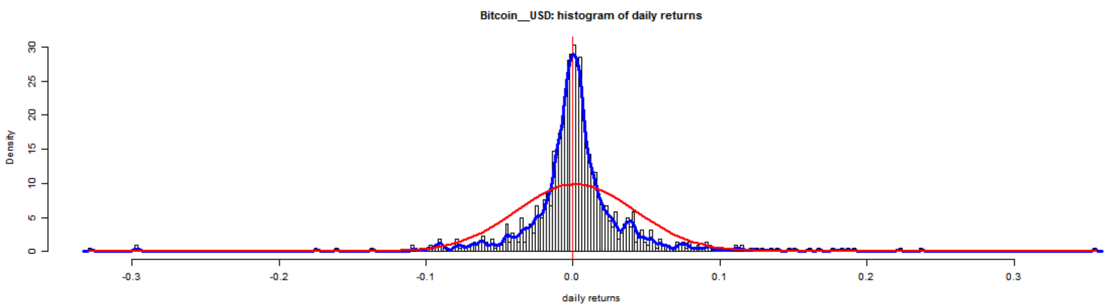

Figure 1: Histogram of Bitcoin/ USD exchange rate with fitted normal distribution and Gaussian KDE overlayed

The next chart 2 shows the qq-plot of the empirical Bitcoin returns versus the quantiles of the standard normal distribution. Again, we observe large deviations from the normal distribution both on the left and the right tail.

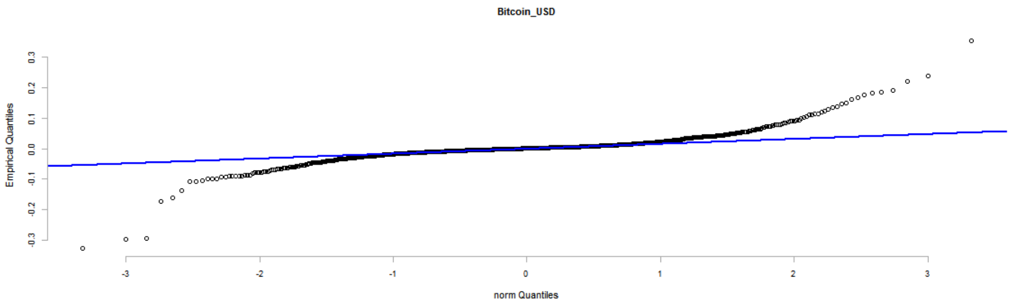

Figure 2: QQ-Plot of the Bitcoin/USD exchange rate versus the normal distribution

3.2 Volatility

From September 2013 until the end of September 2016, the volatility of Bitcoin is about six to seven times larger than the volatility of the G10 currencies. The annualized standard deviation of returns over this period can be seen in the following table 1:

Exchange rate	**Annualized Volatility**
Bitcoin/USD	77%
AUD/USD	11%
CAD/USD	8%
CHF/USD	14%
EUR/USD	9%
GBP/USD	10%
JPY/USD	10%
NOK/USD	12%
NZD/USD	11%
SEK/USD	10%

Table 1: Annualized volatility

In the following figure 3, we have plotted the 90-day rolling volatility (annualized) for Bitcoin and for the exchange rates EUR, JPY, GBP versus the USD. Those currencies represent the four largest G10 currencies.

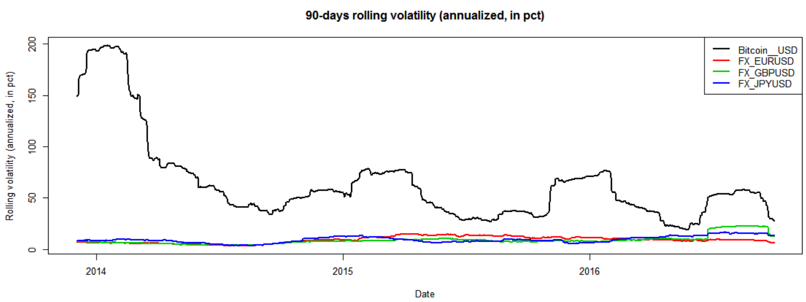

Figure 3: 90-days rolling volatility (annualized, in pct, Bitcoin, GBP, USD, EUR, JPY)

We also show in figure 4 the 90-day rolling volatility of the exchange rate Bitcoin/USD and all G10 currencies. Note that the spike in the CHF/ USD exchange rate volatility in January 2015 is due to the decision of the Swiss National Bank to remove the ceiling of the CHF to the EUR on January 15th, 2015. The downward spike three months later is due to using a 90-day moving window for the calculation of the volatility.

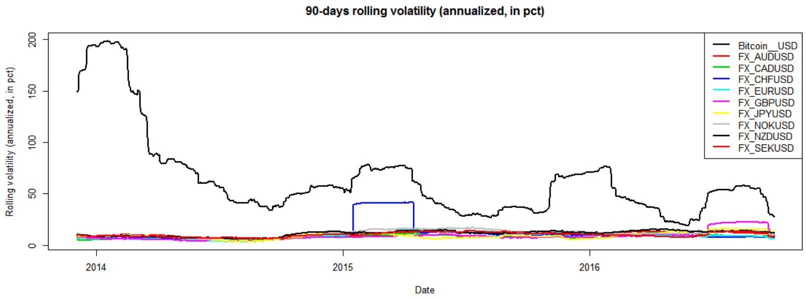

Figure 4: 90-days rolling volatility (annualized, in pct)

So indeed, Bitcoin returns exhibit very high volatility. To put the level of volatility into perspective, note that even during the financial crisis of 2008, fiat currency and equity volatility levels of 70% or more were only ever seen over very short periods of time at the peak of the crisis ((Schwert 2011)). However, as can be seen from figure 4, volatilty of Bitcoin returns has apparently decreased over the last few years - from an almost 200% annualized volatilty during Bitcoin's nascent years down to an annualized volatility of 20% to 30% at the lows in 2016. This development should prove beneficial, as high levels of volatilty typically deter investors (especially institutional ones), and could be one indication that Bitcoin is maturing.

3.3 Extremal index

The extremal index θ is a useful indicator of how much clustering of exceedances of a threshold occurs in the limit of the distribution. For independent data, $\theta = 1$, (though the converse does not hold) and if $\theta < 1$, then there is some dependency (clustering) in the limit. The propensity of of a process to cluster at extreme values is an important property with implications on inference and – in the case of financial time series – risk management.

There are many possible estimators of the extremal index. The ones used here are runs declustering (e.g. (Coles and Coles 2001) , section 5.3.2) and the intervals estimator described in (Ferro and Segers 2003). It is unbiased in the mean and can be used to estimate the number of clusters. In the following table 2 we show the extremal index for the left tail of the returns of the Bitcoin/USD exchange rate and the G10 currencies. This is computed for the 90% quantile, using a run length of four for the runs method.

Exchange rate	Runs declustering			Intervals estimator		
	θ	#clusters	run length	θ	#clusters	run length
Bitcoin/USD	0.487	82	4	0.558	64	3
AUD/USD	0.641	50	4	0.738	57	3
CAD/USD	0.615	48	4	0.776	55	3
CHF/USD	0.654	51	4	0.806	59	2
EUR/USD	0.603	47	4	0.543	40	5
GBP/USD	0.577	45	4	0.489	39	6
JPY/USD	0.500	39	4	0.653	39	4
NOK/USD	0.487	38	4	0.450	34	5
NZD/USD	0.615	48	4	0.836	65	2
SEK/USD	0.718	56	4	0.890	66	2

Table 2: Extremal index for the left tail of Bitoin and the G10 currencies

We can observe that the extremal index using the runs method is the lowest for Bitcoin compared to all the other G10 currencies, with a tie for NOK/ USD. This is an indicator that more clustering of extreme events occurs for Bitcoin than for the traditional fiat currencies. For the intervals method, θ for Bitcoin is at the lower end of all values, but not the lowest. However, those currencies which have a lower value of θ also have a lower estimated

In table 3, we record the extremal index for the positive returns, again at the 90% level and using a run length of four when using the runs method.

Exchange rate	Runs declustering			Intervals estimator		
	θ	#clusters	run length	θ	#clusters	run length
Bitcoin/USD	0.398	45	4	0.418	45	4
AUD/ USD	0.551	43	4	0.573	43	4
CAD/ USD	0.564	44	4	0.616	45	3
CHF/ USD	0.474	37	4	0.456	35	5
EUR/ USD	0.526	41	4	0.419	29	7
GBP/ USD	0.577	45	4	0.781	53	2
JPY/ USD	0.551	43	4	0.474	32	6
NOK/ USD	0.564	44	4	0.583	44	4
NZD/ USD	0.564	44	4	0.586	44	4
SEK/ USD	0.654	63	4	0.874	63	2

Table 3: Extremal index for the right tail of Bitoin and the G10 currencies

We see a similar picture for the right tail than previously for the left tail. Bitcoin seems to exhibit more clustering of exceedances over a threshold.

3.4 Extreme value distributions

Extreme value distributions are distributions characterizing the tails of a distribution. Since we are interested in the risk characteristics of Bitcoin, we are focusing on extreme events, in particular very large negative returns. In extreme value theory, two distributions play an important role: the generalized Pareto distribution as well as the generalized extreme value distribution.

3.4.1 Bitcoin and the generalized Pareto distribution

The generalized Pareto distribution (GPD) is a family of continuous probability distributions. It is used to model the tails of a distribution and specified by three parameters: location μ, scale β, and shape ξ.

Definition 1 (Generalized Pareto distribution) *The cumulative probability distribution function of a generalized Pareto distribution is given by*

$$F_{(\xi,\mu,\beta)} = \begin{cases} 1 - \left(1 + \frac{\xi(x-\mu)}{\beta}\right)^{-\frac{1}{\xi}} & \text{for } \xi \neq 0 \\ 1 - \exp\left(-\frac{x-\mu}{\beta}\right) & \text{for } \xi = 0 \end{cases}$$

for $x \geq \mu$ *when* $\xi \geq 0$, *and* $\mu \leq x \leq \mu - \beta/\xi$ *when* $\xi < 0$, *where* $\mu \in \mathbb{R}$, $\beta > 0$ *and* $\xi \in \mathbb{R}$.

The Pickands–Balkema–de Haan theorem is often called the second theorem in extreme value theory. It gives the asymptotic tail distribution of a random variable X, when the true distribution F of X is unknown. Unlike for the first theorem (the Fisher–Tippett–Gnedenko theorem) in extreme value theory, the interest here is in the values above a threshold.

Theorem 2 (Pickands-Balkema-de Haan) *Let* $(X_1, \ldots, X_n)$ *be a sequence of independent and identically-distributed random variables, and let* F_u *be their conditional excess distribution function. In (Balkema and de Haan 1974) and (III 1975), Pickands, Balkema and de Haan show that for a large class of underlying distribution functions* F, *and large* u, F_u *is well approximated by the generalized Pareto distribution. That is:*

$$F_u(y) \to G_{k,\sigma}(y), \text{ as } u \to \infty$$

where

$$G_{k,\sigma}(y) = 1 - \left(1 + \frac{ky}{\sigma}\right)^{-\frac{1}{k}}$$

if $k \neq 0$.

$$G_{k,\sigma}(y) = 1 - \exp\left(-\frac{y}{\sigma}\right)$$

if $k = 0$. Here $\sigma > 0$, and $y \geq 0$ when $k \geq 0$ and $0 \leq y \leq -\sigma/k$ when $k < 0$.

Since a special case of the generalized Pareto distribution is a power-law, the Pickands–Balkema–de Haan theorem is sometimes used to justify the use of a power-law for modeling extreme events.

We are fitting a GPD distribution to the 150 largest daily negative returns, i.e. the left tail of the distribution of returns. The parameters, together with their standard errors, the threshold value which is implied by those 150 returns as well as the negative log-likelihood is given in table 4. The estimation procedure is using the maximum likelihood method and is based on (Hosking and Wallis 1987).

Exchange rate	ξ	β	ξ **s.e.**	β **s.e.**	**threshold**	**nllh**
Bitcoin/ USD	0.247	0.025	0.1	0.003	0.022	−369
AUD/ USD	−0.221	0.005	0.041	0.000	0.006	−681
CAD/ USD	−0.116	0.003	0.051	0.000	0.004	−717
CHF/ USD	−0.016	0.004	0.082	0.000	0.004	−699
EUR/ USD	0.004	0.004	0.088	0.000	0.004	−679
GBP/ USD	0.197	0.003	0.076	0.000	0.004	−679
JPY/ USD	0.031	0.004	0.075	0.000	0.004	−685
NOK/ USD	−0.032	0.005	0.067	0.000	0.006	−647
NZD/ USD	−0.271	0.006	0.023	0.000	0.006	−666
SEK/ USD	0.030	0.004	0.068	0.000	0.005	−681

Table 4: Maximum likelihood fit of generalized Pareto distribution

We observe that all G10 exchange rates have a much lower shape parameter ξ than the Bitcoin/ USD fit. We show the density of a GPD distribution with the shape parameter $\xi = 0.247$ and the scale parameter $\beta = 0.025$ (Bitcoin/ USD, figure 5) versus the parameters $\xi = 0.004$ and $\beta = 0.004$ (EUR/

USD, figure 6) in the following chart so that the difference between the Bitcoin/ USD exchange rate and most other G10 currencies becomes obvious from a graphical point of view.

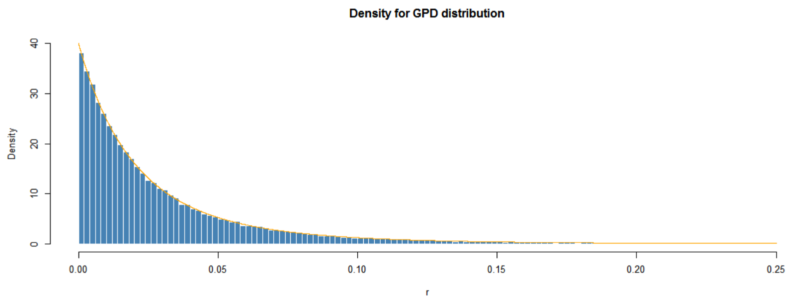

Figure 5: Density of a GPD distribution with parameters $\xi = 0.247$ and $\beta = 0.025$

To understand the differences between Bitcoin and EUR, we also show the density of the GPD distribution fitted to the EUR/ USD exchange rate:

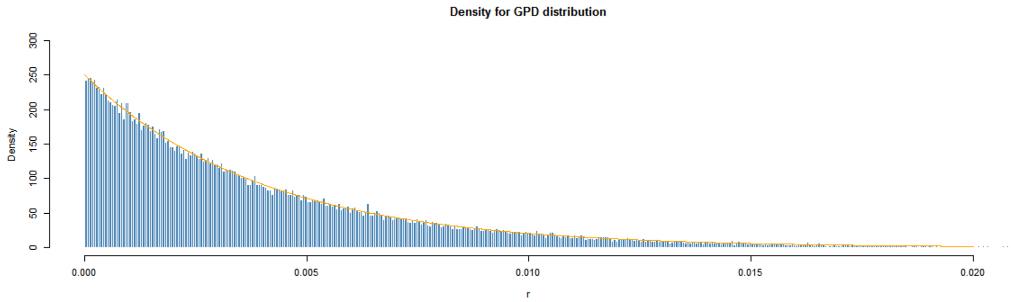

Figure 6: Density of a GPD distribution with parameters $\xi = 0.004$ and $\beta = 0.004$

By comparing both figures, and looking at the scales of the x-axis and the y-axis, we see that the GPD distribution for the Bitcoin/ USD exchange rate has much larger x-values. This is indicative for the Bitcoin exchange rate being much riskier than the EUR/USD exchange rate. For simplicity, we have shown the GPD density chart only for two currencies. With the values in table 4, results for the other G10 currencies are similar to the results when comparing Bitcoin/ USD and EUR/ USD.

We also give the plot of the cumulative excess distribution function $F_u(x-u)$ in figure 7 for the u that corresponds to 150 extremes of the Bitcoin returns. In our case, the threshold u is equal to 0.0224.

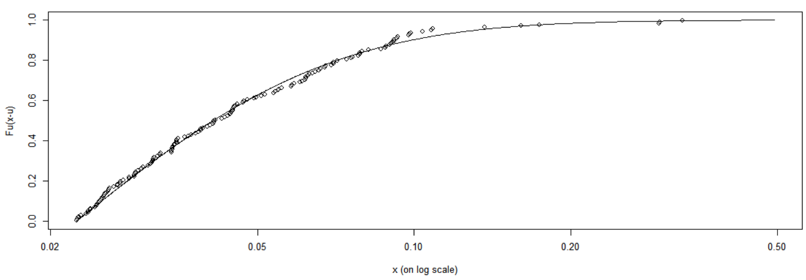

Figure 7: Excess distribution function for Bitcoin

Using the mean excess plot, one can visually decide on the appropriate choice of a threshold. In figure 8 we have plotted the mean excess plot of the negative of the Bitcoin returns, which, for a given threshold u, plots the mean value of all returns exceeding u. We are also showing a vertical line at $u = -0.004$ and the mean excess line given by fitting a GPD distribution to the values exceeding u, obtaining the scale and shape parameters β and ξ with the values $\beta = 0.015$ and $\xi = 0.27$. Once those parameters are obtained, we draw a straight line given by

$$(\beta + \xi \cdot u) / (1 - \xi) \tag{1}$$

(the blue straight line in figure 8). The underlying reason for choosing exactly this line comes from extreme value theory. If we assume that the excess returns follow exactly a GPD distribution with parameters ξ and β, the mean excess would be defined by equation 1. Together with theorem 2, we can approximately assume that the excess returns follow a GPD distribution.

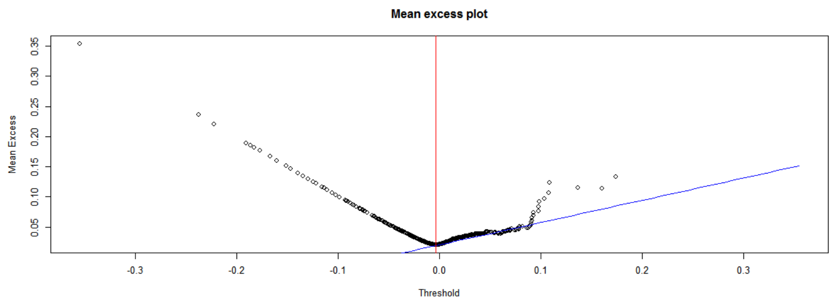

Figure 8: Mean excess plot for the Bitcoin/ USD exchange rate with a vertical line at the threshold $u = -0.004$

An upward trend in the plot shows heavy-tailed behaviour. In particular, a straight line with positive gradient above some threshold is a sign of Pareto behaviour in the tail. A downward trend shows thin-tailed behaviour whereas a line with zero gradient shows an exponential tail.

For comparison, we also show the mean excess plot for the exchange rate between EUR and USD in figure 9. Here, a threshold of $u = 0$ is chosen for the mean excess line. The corresponding parameters are $\beta = 0.004$ and $\xi = 0.054$.

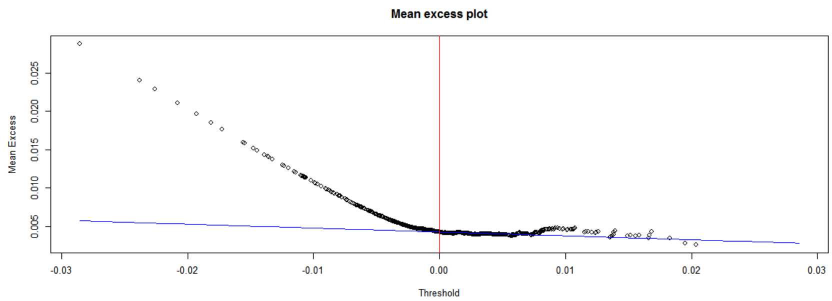

Figure 9: Mean excess plot for the EUR/ USD exchange rate with a vertical line at the threshold $u = 0$

3.4.2 Bitcoin and the generalized extreme value distribution

In probability theory and statistics, the generalized extreme value (GEV) distribution is a family of continuous probability distributions developed within extreme value theory to combine the Gumbel, Fréchet and Weibull families also known as type I, II and III extreme value distributions. The Fisher–Tippett–Gnedenko theorem is a general result in extreme value theory regarding asymptotic distribution of extreme order statistics. The maximum of a sample of iid random variables after proper renormalization can only converge in distribution to one of three possible distributions, the Gumbel distribution, the Fréchet distribution, or the Weibull distribution. Credit for the extreme value theorem (or convergence to types theorem) is given to (Gnedenko 1948).

The role of the extremal types theorem for maxima is similar to that of the central limit theorem for averages, except that the central limit theorem applies to the average of a sample from any distribution with finite variance, while the Fisher-Tippet-Gnedenko theorem only states that if the distribution of a normalized maximum converges, then the limit has to be one of a particular class of distributions. It does not state that the distribution of the normalized maximum does converge. The existence of a limit distribution requires regularity conditions on the tail of the distribution. Despite this, the GEV distribution is often used as an approximation to model the maxima of long (finite) sequences of random variables.

Theorem 3 (Fisher-Tippet-Gnedenko) *Let $(X_1, X_2, \ldots, X_n)$ be a sequence of independent and identically-distributed random variables, and $M_n = \max\{X_1, \ldots, X_n\}$. If a sequence of pairs of real numbers (a_n, b_n) exists such that each $a_n > 0$ and $\lim_{n\to\infty} P\left(\frac{M_n - b_n}{a_n} \leq x\right) = F(x)$ where F is a non-degenerate distribution function, then the limit distribution F belongs to either the Gumbel, the Fréchet or the Weibull family. These can be grouped into the generalized extreme value distribution.*

By the extreme value theorem the GEV distribution is the only possible limit distribution of properly normalized maxima of a sequence of independent and identically distributed random variables.

Definition 4 (Generalized extreme value distribution) *The generalized extreme value distribution has cumulative distribution function*

$$F_{(\mu,\ \sigma,\ \xi)}(x) = \begin{cases} \exp\left\{-\left[1+\xi\left(\frac{x-\mu}{\sigma}\right)\right]^{-\frac{1}{\xi}}\right\} & \text{for } \xi \neq 0 \\ \exp\left(-\exp\left(-\frac{x-\mu}{\sigma}\right)\right) & \text{for } \xi = 0 \end{cases}$$

for $1 + \xi(x-\mu)/\sigma > 0$, where $\mu \in \mathbb{R}$ is the location parameter, $\sigma > 0$ the scale parameter and $\xi \in \mathbb{R}$ the shape parameter. For $\xi > 0$, the support is $x > \mu - \sigma/\xi$, while for $\xi < 0$, it is $x < \mu - \sigma/\xi$. For $\xi = 0$, $x \in \mathbb{R}$

We are fitting a GEV distribution to the block maxima of the negative of Bitcoin and G10 exchange rates versus the USD. The block maxima are taken over periods of one month. The estimated parameters, together with their standard errors, as well as the negative log-likelihood is given in table 5. The estimation procedure is using the maximum likelihood method.

Exchange rate	ξ	σ	μ	ξ **s.e.**	σ **s.e.**	μ **s.e.**	**nllh**
Bitcoin/ USD	0.29	0.028	0.035	0.126	0.004	0.004	-95
AUD/ USD	-0.136	0.003	0.011	0.124	0.000	0.001	-149
CAD/ USD	-0.109	0.003	0.009	0.107	0.000	0.001	-156
CHF/ USD	-0.077	0.003	0.009	0.093	0.000	0.001	-150
EUR/ USD	0.038	0.004	0.009	0.164	0.000	0.001	-139
GBP/ USD	0.254	0.004	0.008	0.112	0.000	0.001	-140
JPY/ USD	0.078	0.004	0.009	0.131	0.000	0.001	-140
NOK/ USD	-0.025	0.006	0.010	0.112	0.001	0.001	-128
NZD/ USD	-0.017	0.003	0.011	0.137	0.000	0.001	-147
SEK/ USD	0.132	0.004	0.010	0.144	0.000	0.001	-141

Table 5: Maximum likelihood fit of generalized extreme value distribution

All G10 exchange rates have a much lower ξ parameter than the Bitcoin/ USD rate. We show the density of a GEV distribution with parameters $\xi = 0.29$, $\sigma = 0.028$ and $\mu = 0.035$ (Bitcoin/ USD, figure

10) versus the parameters $\xi = 0.038$, $\sigma = 0.004$ and $\mu = 0.009$ (EUR/ USD, figure 11) in the following chart so that the difference between the Bitcoin/ USD exchange rate and the G10 currencies becomes obvious from a graphical point of view.

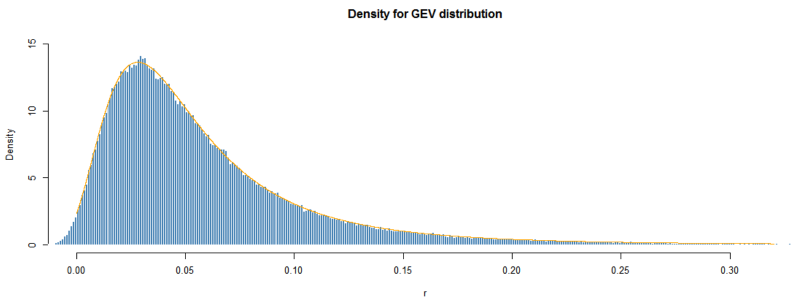

Figure 10: Density of a GEV distribution with parameters $\xi = 0.29$, $\sigma = 0.028$ and $\mu = 0.035$

The density of the GEV distribution fitted to the EUR/ USD exchange rate:

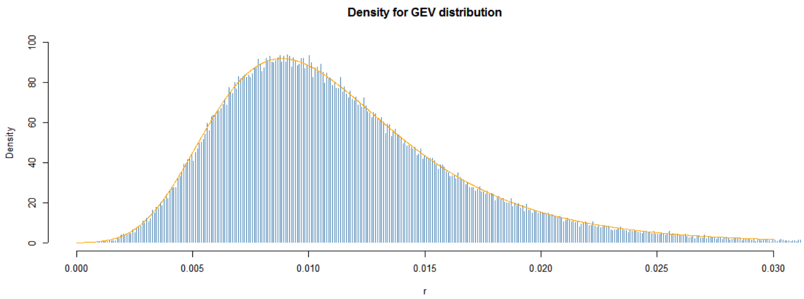

Figure 11: Density of a GEV distribution with parameters $\xi = 0.038$, $\sigma = 0.004$ and $\mu = 0.009$

By comparing both figures, and looking at the scales of the x-axis and the y-axis, we see that the GEV distribution for the Bitcoin/ USD exchange rate has much larger x-values. This is indicative for the Bitcoin exchange rate being much riskier than the EUR/USD exchange rate. For simplicity, we have shown the GEV density chart only for two currencies. With the values in table 5, results for the other G10 currencies are similar to the results when comparing Bitcoin/USD and EUR/USD.

3.5 Value-at-risk and expected shortfall

We want to characterize the losses that can occur in extreme events when investing in Bitcoin. The two most important tail risk measures are value-at-risk and expected shortfall.

Definition 5 (Value at Risk) *Mathematically, if X is a random variable (e.g., the price of a portfolio), then $VaR_\alpha(X)$ is the negative of the α-quantile, i.e.*

$$VaR_\alpha\,(X) = \inf\{x \in \mathbb{R} \mid P(X < -x) \leq 1 - \alpha\} \tag{2}$$

see e.g. (Artzner, Delbaen, Eber, and Heath 1999)

Definition 6 (Expected shortfall) *Expected shortfall estimates the potential size of the loss exceedingVaR_α. For a given random variable X, the expected shortfall is defined as the expected size of a loss that exceeds VaR_α:*

$$E(S_\alpha) = E(X \mid X > VaR_\alpha) \tag{3}$$

(Artzner, Delbaen, Eber, and Heath 1999) argue that expected shortfall, as opposed to value-at-risk, is a coherent risk measure.

In table 6, we record the value-at-risk at the 95% level for Bitcoin and the G10 currencies, using two methods: First, the historical value-at-risk, which corresponds to taking the 95% quantile of the negative exchange rates. Second, the Gaussian value-at-risk, which is computed by first fitting a normal distribution to the data, obtaining the mean μ and standard deviation σ, and then computing analytically the value-at-risk, using the formula

$$VaR_{0.95}^{Gauss} = \mu + \sigma \cdot q_{norm}(0.95)$$

where $q_{norm}(0.95)$ denotes the 95% quantile of the standard normal distribution.

Exchange rate	**Historical VaR**	**Gaussian VaR**
Bitcoin/USD	0.055	0.068
AUD/USD	0.010	0.011
CAD/ USD	0.008	0.008
CHF/ USD	0.009	0.015
EUR/ USD	0.009	0.009
GBP/ USD	0.008	0.010
JPY/ USD	0.010	0.010
NOK/ USD	0.012	0.012
NZD/ USD	0.012	0.012
SEK/ USD	0.010	0.010

Table 6: Value-at-risk for Bitcoin and the G10 currencies

We see that the value-at-risk is about five times larger than the one for the G10 currencies, again showing the substantially higher risk of Bitcoin. You can expect to lose more than 5% in one day, about once every 20 days, when you are invested in Bitcoin. Comparing historical VaR and Gaussian VaR, we can observe that those two numbers are very similar for the G10 currencies but the Gaussian VaR is much higher for Bitcoin. This shows the deviation of the Bitcoin/USD exchange rates from the Gaussian distribution.

In table 7 below, we record the expected shortfall at the 95% level for Bitcoin and the G10 currencies, using two methods: First, the historical expected shortfall, which corresponds to computing the expected shortfall from the historical sample. Second, the Gaussian expected shortfall, which is computed by first fitting a normal distribution to the data, obtaining the mean μ and standard deviation σ, and then computing analytically the expected shortfall, using the formula

$$ES_{0.95} = \mu + \sigma \cdot ES_{norm}(0.95)$$

where $ES_{norm}(0.95)$ denotes the 95% expected shortfall of the standard normal distribution.

Exchange rate	Historical ES	Gaussian ES
Bitcoin/ USD	0.107	0.085
AUD/ USD	0.014	0.014
CAD/ USD	0.011	0.010
CHF/ USD	0.018	0.018
EUR/ USD	0.013	0.012
GBP/ USD	0.012	0.012
JPY/ USD	0.014	0.012
NOK/ USD	0.015	0.015
NZD/ USD	0.015	0.015
SEK/ USD	0.014	0.013

Table 7: Expected shortfall for Bitcoin and the G10 currencies

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We see that the expected shortfall is about eight times larger than the one for the G10 currencies, again showing the substantially higher risk of Bitcoin. Provided you find yourself on one of those 1-in-20 days where you can expect to lose more than 5% in Bitcoin, you will actually end up losing more than 10% on that day.

Comparing historical ES and Gaussian ES, we can observe that those two numbers are very similar for the G10 currencies but the Gaussian ES is lower for Bitcoin. This shows the deviation of the Bitcoin/USD exchange rates from the Gaussian distribution, with Bitcoin being riskier than a comparable normal distribution with the same mean and standard deviation.

4 Conclusion

A detailed understanding of the risk characteristics of Bitcoin is both important for investment and risk management purposes, as well as regulatory considerations. We have characterized the risk properties of the Bitcoin exchange rate versus the USD. By taking data from September 2013 until September 2016 for Bitcoin and the G10 currencies, we could show that Bitcoin returns are much more volatile (albeit with volatility levels decreasing over the course of the last few years), much riskier and exhibit heavier tail behaviour than the traditional fiat currencies. This has implications for risk management, financial engineering (such as bitcoin derivatives) as well as from a regulator's point of view. So for bitcoin to become a mainstream investable asset class, studying these properties is of high importance. To our knowledge, this is the first study looking at the extreme value behaviour of the cryptocurrency Bitcoin.

The volatility of Bitcoin is about six to seven times larger than the one of G10 currencies. In addition, we looked at the tail of the exchange rate returns and fitted both a GPD distribution and a GEV distribution. The fitted parameters for Bitcoin are again substantially different than the ones for the traditional fiat currencies. Using the traditional tail risk measures value-at-risk and expected shortfall, we could quantify that extreme events lead to losses in Bitcoin which are about eight times higher than what we can expect from the G10 currencies. Once every 20 days, you should expect a loss of about 10% on average. The tail index characteristics of Bitcoin also show more clustering of extreme events, both for negative and positive returns, than for traditional fiat currencies.

In summary, we have shown that, apart from other issues, such as security concerns, liquidity issues and other, yet to be solved, problems, Bitcoin, as of now is also a risky investment from a statistical point of view, in particular when looking at the tails of the distribution and extreme events.

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